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# 1 Evaluation of structures affected by Alkali-Silica Reaction (ASR) 2 using homogenized modelling of reinforced concrete

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10

## 11 Abstract

12 The computation of large reinforced concrete structures such as nuclear power plants, dams and  
13 bridges requires realistic behaviour laws to be considered for concrete and reinforcements. ~~The~~  
14 ~~classical way to do this is to use finite element methods in which rebars and concrete are meshed~~  
15 ~~separately.~~ Regarding the problem of cracking in RC structure, meshing separately concrete and rebars  
16 is the classical way to perform a nonlinear finite element analysis. However, when the structures have  
17 to be studied at full scale, the explicit meshing of rebars becomes so heavy that the computing time  
18 reaches values incompatible with engineering applications. The method proposed in this paper consists  
19 of using large finite elements considering reinforcement and concrete as a homogenized material. In  
20 comparison to the mesh reinforcement approach, this one ~~This~~ limits the number of finite elements and  
21 returns to a computation compatible with engineering. The particularity of the proposed model resides  
22 in its ability to treat interaction between rebars and concrete affected by the Alkali-Silica Reaction  
23 (ASR). The model is able to predict the anisotropic swelling induced by the combination of  
24 homogenized rebars and external loadings. An application to a well-documented laboratory test for  
25 reinforced concrete beams shows the ability of the model to assess residual strength capacity of the

26 beam after a long period of ageing in a natural environment. A parametric study of the size of the  
27 finite elements confirms the possibility of using a coarse mesh without loss of the model's predictive  
28 capability.

29 Keywords: Alkali-Silica Reaction, reinforced concrete, finite element

30

31

## 32 1. Introduction

33 Alkali-Silica Reaction (ASR) is an endogenous chemical reaction that damages concrete. It can induce  
34 structural damage as it alters the mechanical strengths of the material. Although the exact reaction  
35 mechanisms are still under discussion in the literature, the main ones have been identified [1] [2] [3].  
36 Through various combined chemical reactions, the alkalis of the pore solution react with the silica  
37 present in amorphous or slightly crystallized phases contained in aggregates, first attacked by hydroxyl  
38 ions. This reaction leads to the production of new phases in the porosity and to swelling of the  
39 concrete. From the mechanical point of view, the action of ASR in concrete can be represented by an  
40 internal pressure. When the stress resulting from this pressure exceeds the local tensile strength,  
41 irreversible cracking is induced. Stresses on the matrix from other sources also play a major role in  
42 expansion as the stress state leads to the development of preferential directions for induced expansions  
43 and, hence, for cracks and damage [4] [5] [6] [7] [8] [9] [10]. Such ASR damage modifies the bearing  
44 capacity of the affected structures and modelling is necessary to evaluate the structural safety. The  
45 characterization of ASR-damage and the modelling of its impact can also be used to optimize and  
46 facilitate repairs and rehabilitation works.

47 Constitutive models are generally established after experimental and laboratory studies and may be  
48 validated by confrontation with results obtained on laboratory structures. Hence, the use of such  
49 models for real damaged structures may be questionable due to the differences between ASR  
50 mechanisms in laboratory conditions and in the actual environment of the structure concerned. **Due to**

51 the difference between ASR scale and structural scale, the establishment of multi-scale approaches  
52 constitutes a topical and a complex issue. These last years, efforts have been done towards the multi-  
53 scale model development [11] [12] [13]. Hence, different approaches can be found in the literature.  
54 According to [14], they can be distinguished in different categories: models based on concrete  
55 expansion, models based on internal pressure, models based on gel production and model based on  
56 ions diffusion reaction Note that in this paper, the ASR modelling belongs to the category of gel  
57 production types. Further complexity may be introduced if the structures are highly reinforced.  
58 Reinforcements in ASR-affected structures cause restrained expansion, chemical prestressing and  
59 oriented cracking. For numerical simulations, this can lead to several difficulties. ASR expansion  
60 restrained by the reinforcement steel leads to the development of stress concentration in the steel-  
61 concrete interface zone. For modelling based on damage theory, such stress localized in small zones of  
62 the mesh can induce an overestimation of the damage and a total loss of bonding between the concrete  
63 and the reinforcement bar. However, recent experimental work [15] indicates that this modelled  
64 phenomenon may not be realistic. Because of this unrealistic evaluation of stress at the interface  
65 between concrete and steel, the whole mechanical behaviour of the structure is misrepresented. In  
66 order to avoid such numerical complexities, and limit computational times, reinforcement can be  
67 modelled by assuming reinforced concrete to be a single material containing concrete and rebars,  
68 where the reinforcement bars do not have to be specifically meshed [16] [17] [18]. The steel  
69 contribution is evaluated along the directions of reinforcements through their own behavioural laws.  
70 The reinforced concrete response is then assessed by a mixing law combining steel and concrete  
71 contributions, according to the concrete/steel ratio. From the application point of view, and  
72 comparatively with approach needing the meshing of the reinforcements, such a homogenization  
73 approach can lead to significant time saving, particularly for highly reinforced structures. As  
74 reinforcement does not have to be explicitly meshed, the structures can be evaluated with large sized  
75 elements, thus reducing computation times without significant loss of prediction capability.

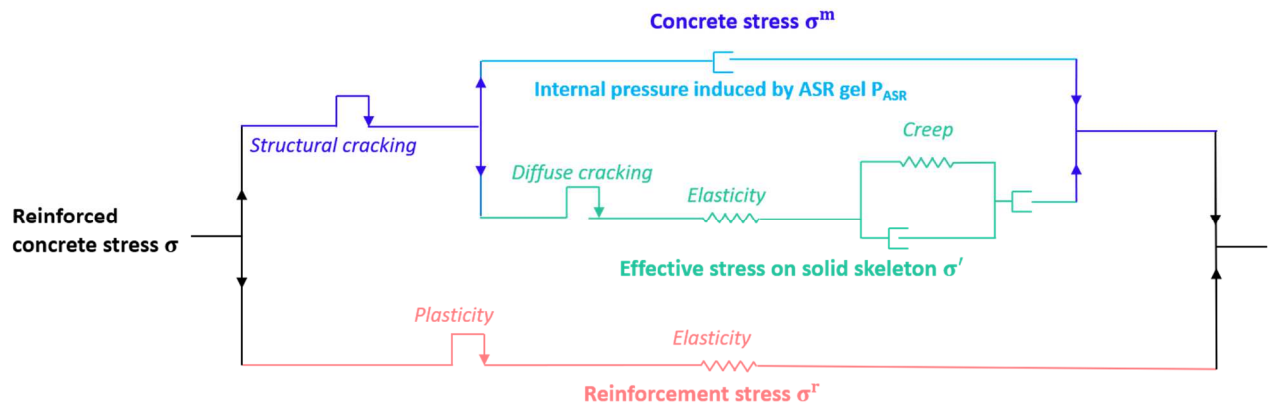
76 The aim of this paper is to validate the suitability of combining ASR-modelling with the assumption of  
77 homogenized reinforcement, by comparing model predictions with experimental results from the

78 literature. This comparison is performed for the two main phases of the service life of the structures:  
79 the ageing period (development of expansion) and the final bending, so as to assess the capacity of the  
80 model to represent both the Service Limit State (SLS) and the Ultimate Limit State (ULS) of the  
81 structure. It is also important to verify the dependence of the model on the mesh size in such  
82 modelling. This is the second objective of this paper. It is attained through a parametrical study on the  
83 sizes of the finite elements.

84 The main features of the model used in this work are presented first. Then the model is validated  
85 through a comparison with the experimental works performed by Ohno et al. [19] on the mechanical  
86 behaviour of beams affected by ASR. During the period of aging, the beams were subjected to natural  
87 weather, with small moisture gradients, to allow ASR advancement to develop without laboratory  
88 acceleration. After this, they were subjected to a four-point bending test until failure. Due to the  
89 induced chemical prestressing, the strain during the ageing phase and the flexural response of reactive  
90 beams were different from those of non-reactive beams. Strain evolution during the ageing period is  
91 analysed, as are the load/displacement curves and the crack patterns obtained during the simulation of  
92 the bending test. The sensitivity of the model to the mesh size is discussed for the two periods.

## 93 2. Constitutive model

94 In the present study, the reinforced concrete structure is modelled through a homogenized approach in  
95 which the reinforced concrete matrix is considered as a single material. The contributions of the  
96 concrete and the steel reinforcement are evaluated separately before being combined in a [homogenized](#)  
97 [law single behaviour law](#) according to their relative quantities. ~~The global mechanical scheme~~  
98 ~~combining concrete and steel modelling is presented in~~( Figure 1)



99

100 *Figure 1: Rheological scheme: concrete and reinforcement modelling combined in a [single reinforced concrete](#)*  
 101 *[behaviour law](#)*

102 The different terms defined in Figure 1 are combined by a global Equation (17) which will be  
 103 presented after the definition of each term. In the first section, the ASR pressure and damage are  
 104 presented and in the following some aspects of concrete and reinforcement behaviours are specified.

105 To model the internal expansion induced by ASR, the poromechanical framework presented in [20]  
 106 and [21] is used. This theory allows the solid phases of the matrix to be differentiated from the  
 107 interstitial ones. The concrete stress  $\sigma^m$  is split in two parts: the internal pressure  $P_{ASR}$  due to the new  
 108 phases produced by ASR, and the effective stress  $\sigma'$  applied to the solid skeleton of the concrete  
 109 matrix (Figure 1).

110 Considering the reduction of mechanical properties due to ASR or external loading, the model is also  
 111 defined according to the damage theory of [22]. In this framework, the effective stress is defined with  
 112 respect to the [healthy sound](#) part of the concrete. Hence, the effective stress in the damage sense is the  
 113 concrete stress  $\sigma^m$ . Therefore, the total induced stress is the mean stress on the total surface of the  
 114 material. The relation between effective and total stress is obtained from the evaluation of damage.

115 In concrete structures, the cracks due to external loading or to strain gradients are localized. They are  
 116 called 'structural cracking' in this paper. For ASR-affected structures, such structural cracks ~~can~~ [must](#)  
 117 be distinguished from material cracks due to internal expansion caused by ASR at the aggregate scale:  
 118 [The formulation of the model distinguishes the two types of cracking with two types of criteria.](#)

119 Structural cracks are induced by the "total" principal stresses, while the diffuse cracking due to ASR  
 120 starts in the reactive aggregates. The stress induced by the gel pressure can be possibly balanced by  
 121 macroscopic compressive stress (which induces anisotropic cracking and then anisotropic swelling).  
 122 Material cracking due to ASR is called 'diffuse cracking' here, as in Figure 1. In the present model,  
 123 the distinction between the two types of cracking is made by two different criteria. At macroscopic  
 124 scale, a smeared crack approach [23] is used to manage the structural cracking. ~~For the structural~~  
 125 ~~cracking at macroscopic scale, e.~~ Compressive and shear effects are managed by a non-associated  
 126 Drucker-Prager criterion that controls the evolution of the corresponding plastic strains. Tensile  
 127 macroscopic effects are driven by three associated orthogonal Rankine criteria (in the main directions  
 128 of tension). For the diffuse cracking, a same kind of Rankine criteria are used but written in terms of  
 129 the main poromechanical effective stresses.

130 Once ASR cracking is initiated, it reduces the mechanical characteristics. Different damage variables  
 131 are used in the model (according to external/internal phenomena). As this paper focuses on assessing  
 132 reinforced concrete structures damaged by ASR, only some aspects of the model are presented here.  
 133 All the other aspects of the model can be found in [16].

## 134 2.1. Alkali-Silica Reaction

### 135 2.1.1. Advancement

136 To model the ASR induced swelling, the advancement must first be assessed. More than cut-off limits  
 137 for the reaction to take place, the environmental conditions strongly impact the evolution of its  
 138 kinetics. Hence advancement evaluation must take them into account. Note that, in the present work,  
 139 the variation of the saturation degree in the concrete is assumed to be negligible.

140 Thus, ASR advancement degree  $A^{ASR}$  is defined by the following equation, based on Poyet's work  
 141 [24] [25]:

$$\frac{\partial A^{ASR}}{\partial t} = \frac{1}{\tau_{ref}^{ASR}} C^{T,ASR} \cdot \frac{(1 - A^{ASR})}{+} (1 - A^{ASR}) \quad (1)$$

142 where  $\tau_{ref}^{ASR}$  is a material parameter driving the ASR kinetics, calibrated at the absolute reference  
 143 temperature  $T_{ref}$ .  $e^{\frac{E^{ASR}}{R}} C^{T,ASR}$  is the coefficient that modifies the kinetics according to the  
 144 temperature.

145 Larive [26] highlighted the non-linear impact of temperature on ASR-kinetics. It can be characterized  
 146 by an Arrhenius law, with an activation energy,  $E^{ASR}$ , estimated at 40 kJ/mol:

$$C^{T,ASR} = \exp\left(-\frac{E^{ASR}}{R}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right) \quad (2)$$

### 147 2.1.2. Effective gel of ASR

148 In most of experimental studies, ASR expansion evolves according to three phases: a latency period,  
 149 an acceleration of the expansion and, finally, a strain plateau. Several mechanisms can explain the  
 150 latency period. Physical considerations include the time necessary for ASR phases to induce sufficient  
 151 pressure to cause internal cracks [27] [28]. Chemical considerations can also explain the latency period  
 152 by the succession of chemical reactions according to the chemical equilibrium in the pore solution [3].  
 153 Whatever the reason, poromechanical modelling has to be able to reproduce this latency time during  
 154 which chemical reaction starts without inducing expansion. It can be considered through the notion of  
 155 effective volume of gel:

- 156 - As long as the advancement is lower than a threshold, the phases produced by ASR do not  
 157 induce effective expansion,
- 158 - Once this advancement threshold is reached, the expansion begins.

159 It can be obtained by the following equations:

$$\Phi_{ASR}^{eff} = \Phi^{ASR,\infty} \frac{(A(t) - A_{LAT})}{(1 - A_{LAT})} \text{ if } A(t) > A_{LAT} \quad (3)$$

$$\Phi_{ASR}^{eff} = 0 \quad \text{if } A(t) < A_{LAT}$$



160 with  $\Phi^{ASR,\infty}$  the maximal volume of ASR-phases and  $A_{LAT}$  the advancement threshold below which  
 161 the phases do not induce expansion. This threshold is a fitting parameter which should find its physical  
 162 origin in the nature of the reactive aggregate and the cement paste properties. All the parameters  
 163 delaying expansion lead to increases in the advancement threshold.

### 164 2.1.3. Internal pressure due to ASR

165 In the poromechanics framework, the action of ASR can be represented by the internal pressure,  $P_{ASR}$ ,  
 166 which acts in the porosity of the concrete and is evaluated from the volume of effective gel,  $\Phi_{ASR}^{eff}$  :

$$P_{ASR} = M_{ASR} \cdot \langle \Phi_{ASR}^{eff} - \langle \Phi_{ASR}^{v,eff} + \Phi_{ASR}^{\Delta\phi} + \Phi_{ASR}^{pl} \rangle^+ \rangle^+ \quad (4)$$

167 In Equation (4), the term  $\Phi_{ASR}^{v,eff}$  represents the volumetric proportion of ASR products that can  
 168 penetrate into the available porosity around the reactive sites according to the pressure. This  
 169 dependence of the pressure on available porosity is necessary to model expansion under triaxial  
 170 stresses [7]. This term corresponds to the proportion of ASR products by volume in the available  
 171 porosity; it can increase with the pressure as explained in [7].

172 However, unrealistic responses appeared when the pressure of ASR products on the concrete  
 173 decreases. External loading (or unloading after compression) can lead to a positive strain resulting in  
 174 an increase in concrete porosity by deformation of the surrounding cement matrix and, thus, in a  
 175 decrease of the pressure. As the volumetric proportion of ASR-products in the available porosity,  
 176  $\Phi_{ASR}^{eff}$ , was proportional to the internal pressure, the pressure decrease induced by the decompression of  
 177 concrete led to the return of gel to its initial reactive site, slowing down the pressure decrease (which  
 178 then kept a value high enough to provoke new cracks when the external compression stress was  
 179 removed).

180 Such a phenomenon is not realistic because the ASR products partially crystallize (particularly in  
 181 cement paste in presence of calcium ions [3]) and cannot return to the reactive site once formed.  
 182 Therefore, the model was modified. In the present version, the modelling assumes that, once formed in  
 183 aggregate or concrete porosity, the ASR-products can no longer move: the volume of ASR-products in

184 the available porosity stays in the porosity even if the pressure decreases. Thus,  $\Phi_{ASR}^{v,eff}$  is calculated  
 185 using the maximum value of the ASR pressure found during the whole life of the structure,  $P_{ASR}^{MAX}$ ,  
 186 instead of the current value of  $P_{ASR}$ :

$$\Phi_{ASR}^{v,eff} = \Phi_{ASR}^v \cdot \frac{P_{ASR}^{MAX}}{R_t} \quad (5)$$

187 This assumption is a simplification. The real behaviour is probably a combination of two phenomena:  
 188 the part of a product that is not yet crystallized can probably still move in the porosity while the  
 189 crystallized part is fixed. To illustrate this new particularity of the model, the initial and current  
 190 hypotheses are both presented in Figure 2.

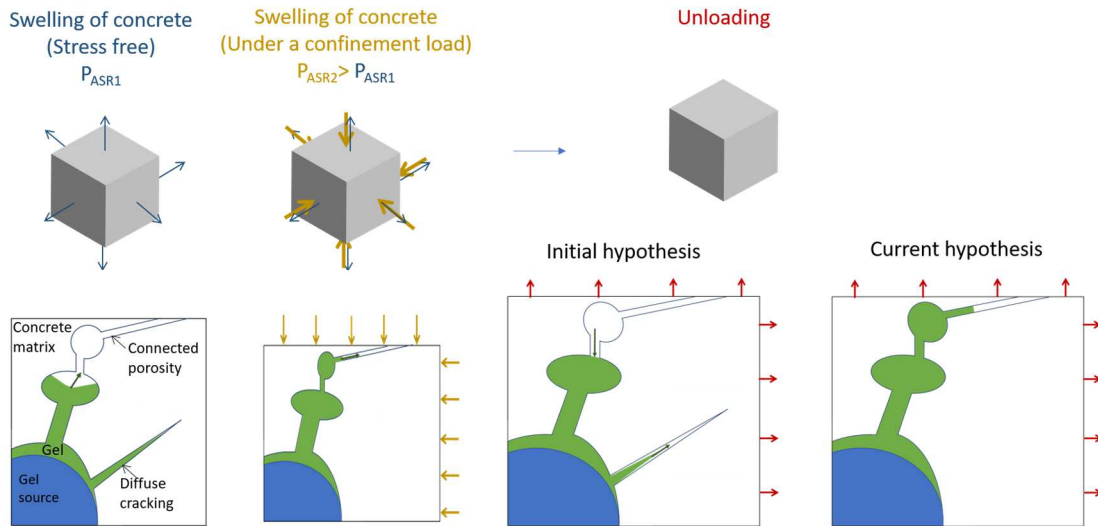


Figure 1: Evolution of ASR gel in connected porosity in a stress-free state compared to a load/unload state; initial and current model hypotheses

191  
 192 As volume strain causes the volume of porosity to vary, if the volume strain is positive, the pressure of  
 193 ASR phases decreases (and conversely, the pressure increases if the volume strain is negative). The  
 194 term  $\Phi_{ASR}^{\Delta\phi}$  of Equation (4) represents the variation of the porosity volume with volume strain, whatever  
 195 the strain origin (elastic response of the material  $\epsilon^{el}$  or delayed response due to creep  $\epsilon^{cr}$ ).

$$\Phi_{ASR}^{\Delta\phi} = b_{ASR} \cdot tr(\epsilon^{el} + \epsilon^{cr}) \quad (6)$$

196 For the sake of simplicity and because of a lack of precise quantitative data on the filling of cracking  
 197 induced by ASR, the model assumes the total filling of diffuse cracking,  $\varepsilon^{pl,ASR}$ , by ASR phases (term  
 198  $\Phi_{ASR}^{pl}$  of Equation (4)). In the approach presented in the present paper, the diffuse cracking due to ASR  
 199 is modelled by plastic strain, so this term is written as:

$$\Phi_{ASR}^{pl} = tr(\varepsilon^{pl,ASR}) \quad (7)$$

200 In Equation (4) and Equation (6),  $M_{ASR}$  and  $b_{ASR}$  are respectively the Biot modulus and the Biot  
 201 coefficient related to ASR in concrete. They quantify the mechanical impact of the interactions  
 202 between the different phases (ASR phases / aggregate and concrete) according to their respective  
 203 rigidities.

204 The Biot coefficient is evaluated according to the effective volume of gel  $\Phi_{ASR}^{eff}$  by a homogenization  
 205 method [29]:

$$b_{ASR} = \frac{2\Phi_{ASR}^{eff}}{1 + \Phi_{ASR}^{eff}} \quad (8)$$

206 The Biot modulus is then assessed from  $b_{ASR}$  and from the volume stiffness of the concrete matrix,  
 207  $K_S$ , and of ASR products,  $K_{ASR}$ , according to the Biot theory [20,21]:

$$\frac{1}{M_{ASR}} = \frac{b_{ASR} - \Phi_{ASR}^{eff}}{K_S} + \frac{\Phi_{ASR}^{eff}}{K_{ASR}} \quad (9)$$

## 208 2.1.4. ASR-diffuse cracking

### 209 2.1.4.1. Criteria

210 When the internal pressure induced by ASR exceeds the local effective tensile strength,  $\widetilde{R}_I^t$ , ASR  
 211 diffuse cracking is initiated. To drive this mechanism, an anisotropic associated Rankine criterion  
 212 written in poromechanical terms is used:

$$f_I^{ASR} = \tilde{\sigma}_I^{eq} - \widetilde{R}_I^t \quad \text{with } I \in [I, II, III]$$

$$\text{and } \tilde{\sigma}_I^{eq} = P_{ASR} + \min(\tilde{\sigma}_I, 0) \quad (10)$$

213 where  $\tilde{\sigma}_I$  is the stress in the principal direction I.

214 Once cracking is initiated, the crack opening is represented by the plastic strain,  $\varepsilon_I^{pl,ASR}$ , and an  
 215 increasing pressure is needed to propagate cracking. To represent this requirement, the tensile strength  
 216 used in Equation (10) increases according to a linear hardening law. ~~The hardening law has been~~  
 217 ~~calibrated in previous numerical works [30] based on experimental results [4].~~ The hardening was  
 218 close to 3% of the concrete Young's modulus. This small hardening is needed to provoke stable  
 219 propagation of cracking. If its value tends to zero it is no longer possible to control the isotropy of free  
 220 swelling because the cracking could appear arbitrarily in any principal direction and propagate to limit  
 221 the rise of pressure in the gel, which, in turn, would be unable to initiate cracking in the other principal  
 222 directions.

223 If a direction of the swelling concrete is reinforced [31] [8] [32] [33] [19], a chemical prestressing  
 224 phenomenon occurs and leads to a small expansion along the reinforcement direction (the decrease of  
 225 swelling amplitude in this direction is greater than the elastic effect of the steel bar alone [31]) and to a  
 226 compressive stress (Figure 3-a and b). ~~If the expansion is decreased in one direction (due to~~  
 227 ~~compressive stress), the effect of pressure in the model leads to greater expansion in the free~~  
 228 ~~directions~~, as observed for loaded specimens with a stress-free direction [26] [34] [35] [5] (Figure 3-a).  
 229 The damage is smaller in the reinforced direction than for the stress-free specimen but is slightly  
 230 greater in the direction perpendicular to reinforcement, due to the pressure rise induced by the  
 231 hardening law (Figure 3-c).

232 ~~The strain anisotropy is a consequence of stress state induced by reinforcement, and the anisotropy of~~  
 233 ~~stress state on swelling. The hardening law which manages this report phenomenon has been~~  
 234 ~~calibrated in previous numerical works [30] based on experimental results from [4].~~

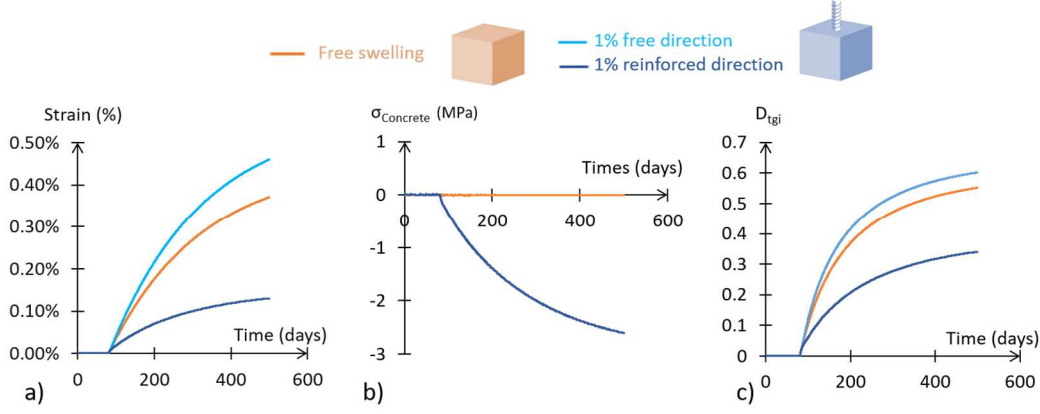


Figure 2: Effect of reinforcement compared to stress-free swelling, a) Strain evolution, b) Concrete stress evolution, c) ASR-damage evolution

235

#### 236 2.1.4.2. Evolution of mechanical properties with ASR

237 As far as the mechanical strength degradation is concerned, the reduction of tensile strength and  
 238 modulus is usually observed in most experimental studies [36] [37] [4] [38] [39]. The impact on the  
 239 compressive strength is smaller and is more dependent on the concrete composition and test conditions  
 240 [36] [40] [38] [5] [39] [37] [41].

241 In the present model, the damage induced by the swelling is anisotropic and assessed from the plastic  
 242 strain obtained by the previous ASR-cracking criterion:

$$D_I^{t,ASR} = \frac{\varepsilon_I^{pl,ASR}}{\varepsilon_I^{pl,ASR} + \varepsilon^{k,ASR}} \quad (11)$$

243 where  $\varepsilon^{k,ASR}$  is a characteristic strain evaluated at about 0.3% for the ASR [42].

244 The damage in compression is smaller, especially because of crack reclosure. It is evaluated from the  
 245 combination of the tensile damage in the two orthogonal directions (Equation (12)). (See [43] [44] for  
 246 further explanations on the concept and on its applicability to the ASR phenomenon.)

$$D_I^{c,ASR} = 1 - \left( (1 - D_{II}^{t,ASR})(1 - D_{III}^{t,ASR}) \right)^{\alpha_{ASR}} \quad (12)$$

with  $\alpha_{ASR} = 0.15$

247 The mechanical part of the model can also take the mechanism of swelling induced by the delayed  
 248 ettringite formation (DEF) into account. Although the evaluation of the chemical advancement may  
 249 differ, the mechanical consequences of induced pressure are close [45] and, thus, can be driven by the  
 250 same equations [29]. Moreover, the mechanical part of the model has been validated in the ASR and  
 251 DEF-context by the work of Morenon on laboratory reinforced beams with meshed reinforcement bars  
 252 and non-reinforced dams [46]. The present study focuses on the interest of homogenized reinforced  
 253 concrete in the context of ASR.

## 254 2.2. Concrete

255 It is important to note the difference between diffuse and structural cracking. Structural cracking can  
 256 be induced by external loading, or by a strong strain gradient. Unlike cracks induced by ASR,  
 257 structural cracks are localized. Different plastic criteria manage tensile and shear cracking. As this  
 258 cracking occurs at the structural scale, their characterizations do not need the poromechanical  
 259 framework. Thus, structural criteria are written in terms of total stresses instead of the poromechanical  
 260 effective stress. By using these different kinds of criteria, the two type of cracking evolve  
 261 independently. Finally, the total concrete stress  $\sigma_{ij}^c$  is assessed by the combination of the diffuse and  
 262 the structural cracking as in:

$$\begin{aligned}\sigma_{ij}^c &= (1 - D_{ASR})(1 - D_{structural})\widetilde{\sigma}_{ij}^c \\ \widetilde{\sigma}_{ij}^c &= \widetilde{\sigma}_{ij}^{c'} - \delta_{ij}(b_{ASR}P_{ASR})\end{aligned}\tag{13}$$

263 The ASR damage, Biot coefficient and pressure have been already explained in Equations (11)-(12),  
 264 Equation (8) and Equation (4) respectively.  $\delta_{ij}$  is the Kronecker delta symbol, which is equal to one  
 265 if  $i=j$  and zero otherwise. The evaluation of structural damage is presented in the following lines.

266 In tension, pre-peak and post-peak damage are considered. The first is isotropic, and evaluated from  
 267 the mechanical characteristics of the concrete: its Young's modulus,  $E$ , its tensile strength,  $R_t$ , and the  
 268 value of strain at the tensile peak  $\varepsilon_{pic}^t$  (Equation (14)).

$$D_{pre-peak}^t = 1 - \frac{R_t}{E \varepsilon_{pic}^t} \quad (14)$$

269 After the peak, if loading is anisotropic, damage becomes anisotropic too. Structural cracking is then  
 270 managed by a combination of plasticity and damage theories. Plastic strains are assessed through an  
 271 anisotropic associated Rankine criterion (15). If pre-peak tensile damage exists, it is taken into  
 272 consideration through a reduction of the tensile strength used in the following equations:

$$f_I^t = \tilde{\sigma}_I - \tilde{R}_I^t \text{ with } I \in [I, II, III] \quad (15)$$

$$\text{With } \tilde{R}_I^t = \frac{R_t}{1 - D_{pre-peak}^t}$$

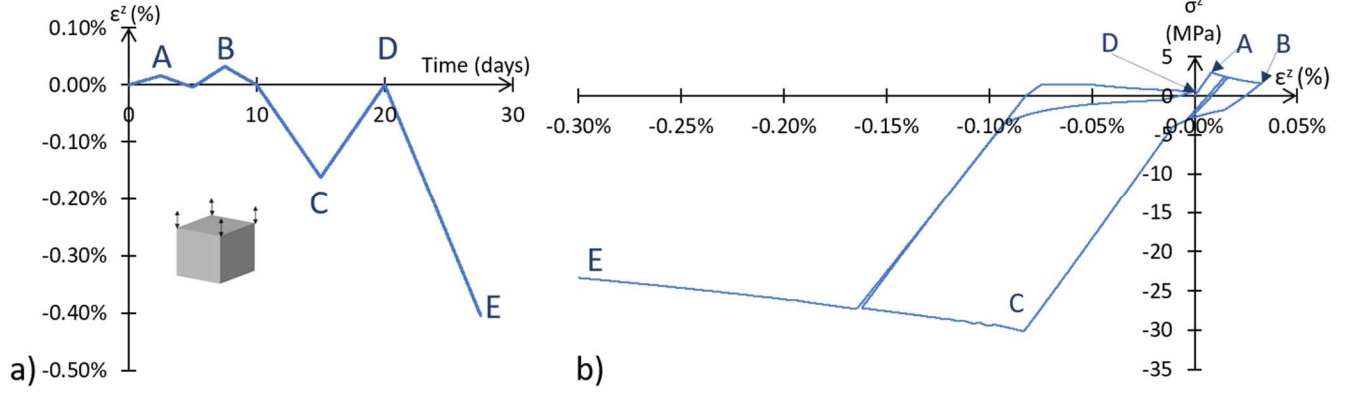
273 The anisotropic induced damage is:

$$D_I^t = 1 - \left( \frac{w_I^{k,t}}{w_I^{k,t} + w_I^{pl,t,max}} \right)^2 \quad (16)$$

274 where  $w_I^{k,t}$  is the characteristic crack opening corresponding to the fracture tensile energy  $G_{ft}$  and  
 275  $w_I^{pl,t,max}$  is the maximal crack opening obtained with the value of plastic strain and element size in  
 276 the principal tensile direction.

277 To be independent of the mesh size, an energy regularization based on the Hillerborg method is used.  
 278 The approach consists of using the maximal distance between two nodes of the finite element in the  
 279 considered direction as the dissipation length (details can be found in [47]).

280 The model can also consider structural damage in shear and compression, and even damage induced  
 281 by crack reclosure. These parts of the model are not described here but can be found in [16]. To  
 282 illustrate these behaviour laws, an example of a uniaxial cyclic load is given in Figure 4.



283

284 *Figure 4 : Uniaxial-tension-compression loads with damage, ( $R_t=3MPa$ ,  $R_c=30MPa$ ), a) Imposed strain versus*  
 285 *time, b) Model response*

### 286 2.3. Reinforcement

287 The aim of the homogenized approach is to avoid explicit meshing of the reinforcement. The material  
 288 is directly a ‘reinforced concrete’ without distinction between rebars and concrete in the mesh. Several  
 289 directions of reinforcements can be considered in each finite element. Each steel rebar is defined by its  
 290 direction (a vector field) and its surface ratio  $\rho^r$  (a scalar field expressed as the ratio between steel and  
 291 concrete cross-sections). The mechanical properties of the reinforcement (Young’s modulus  $E^r$ , limit  
 292 of elasticity  $f_y^r$ , hardening coefficient  $H^r$ ) form the input data.

293 The mechanical behaviour of the reinforcement is driven by an elastoplastic law controlled by a  
 294 kinematic uniaxial plastic hardening criterion (17):

$$f^r = |\sigma^r - H^r \epsilon^{r,pl}| - f_y^r$$

$$\text{with } \sigma^r = E^r (\epsilon^r - \epsilon^{r,pl}) \quad (17)$$



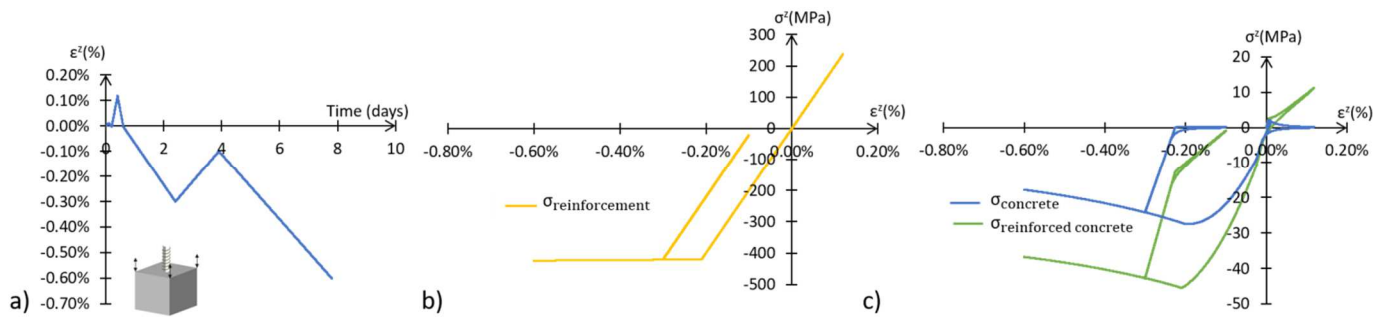
295 2.4. Combination of concrete and reinforcement

296 The stress in homogenized reinforced concrete,  $\sigma_{ij}$ , is the combination of the stress in the concrete,  $\sigma_{ij}^c$   
 297 (assessed by Equation 13), and that in the reinforcement,  $\sigma_{ij}^{r,n}$  according to the steel/concrete ratio,  
 298  $\rho^{r,n}$ :

$$\sigma_{ij} = \left( 1 - \sum_{n=1}^{Nr} \rho^{r,n} \right) \sigma_{ij}^c + \sum_{n=1}^{Nr} \rho^{r,n} \sigma_{ij}^{r,n} \quad (18)$$

299 with  $Nr$  the number of types of reinforcements. As in the previous part, an example of a uniaxial  
 300 cyclic load is given to illustrate this behaviour law in Figure 5.

301 Homogenized reinforced concrete based on this approach has already been used for the evaluation of a  
 302 nuclear power plant containment wall without ASR [48].



304 Figure 5 ; Uniaxial-tension-compression loads with damage, ( $R_t=3MPa$ ,  $R_c=30MPa$ ), a) Imposed strain versus  
 305 time, b) Reinforcement stress, c) Concrete and reinforced concrete stress

306 3. Study case

307 3.1. Details of the experiments used for the validation [19]

308 The model is now used to reproduce the results obtained in the experimental study performed by Ohno  
 309 et al. [19] on the structural behaviour of reinforced concrete beams affected by ASR. To consider  
 310 different levels of ASR degradation, the beams were tested to failure after two ageing phases, of 17

311 and 45 months. Four beams were monitored: two reactive and two non-reactive beams. After the  
 312 ageing period under natural weather, the beams were loaded to failure by a four-point bending test.  
 313 Transversal reinforcement was ensured by 10 mm diameter bars, and longitudinal reinforcement by 4  
 314 x 25 mm diameter bars. The location of the reinforcement is shown in Figure 6.

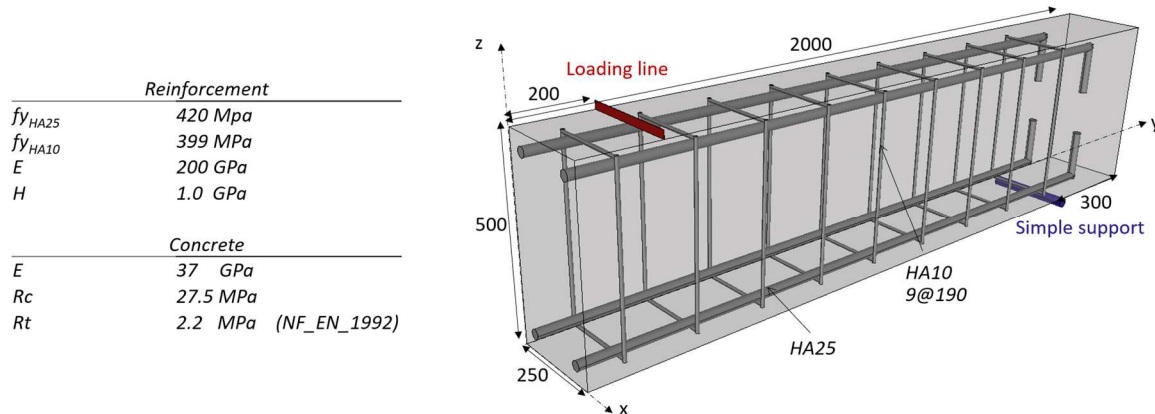


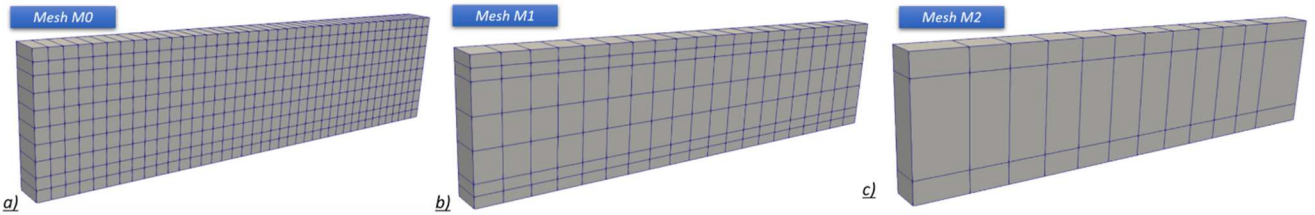
Figure 3 : Mechanical characteristics and half-beam reinforcement (unit = mm)

315  
 316 The compressive strength and Young's modulus of the two concretes (sound and reactive) used for the  
 317 simulation of the beams were supplied by the authors of the experimental programme. The concrete  
 318 properties used in the numerical evaluation are given in Figure 6. They are representative of concrete  
 319 after long cement hydration without ASR damage (properties of the non-reactive concrete at 17  
 320 months). For simplicity, the gain of mechanical properties by hydration, which can partly counteract  
 321 ASR damage [49], is not computed in this study and the impact of ASR on mechanical properties is  
 322 directly evaluated by the model during expansion (see part 2.1.4.2). Due to the difference between  
 323 material types (mortar or concrete), the mechanical properties and the value of mechanical parameter  
 324 ( $R_t$ ,  $R_c$ ,  $E$ ,  $G_{fit}$  ...) can be different. Hence, these differences are directly considered by the input data  
 325 of the model.

### 3.2. Mesh used in the numerical analysis

327 In order to validate the ability of the model to treat large finite elements with the homogenized  
 328 behaviour law of reinforced concrete affected by ASR, the numerical study uses three mesh sizes: M0

329 is the finest mesh and M2 the coarsest (Figure 7). The modelling was performed with 8 nodes cube  
 330 elements with linear interpolation functions.

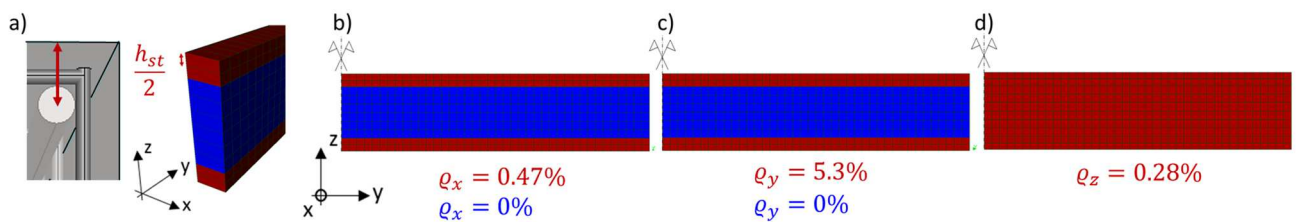


331  
 332 Figure 7 : a) The finest mesh M0, b) The intermediate mesh M1, c) The coarsest mesh M2

333 To obtain a realistic structural response in bending, homogenized finite elements were designed to  
 334 respect the real positions of reinforcements barycentre. Figure 8 presents the mesh M0 with the ratios  
 335 of homogenized reinforcements used in the three directions:

- 336 - the transversal and the longitudinal reinforcements are in the upper and lower parts of the  
 337 beam, around the position of the centre of gravity of the longitudinal bars (Figure 8-a, b and  
 338 c),
- 339 - To avoid making the mesh too sophisticated, vertical reinforcements are homogenized along  
 340 the whole length of the beam (Figure 8-d).

341 In the model, the reinforcement ratio is obtained by dividing the area of the steel reinforcement by the  
 342 cross-section of the area of the homogenized reinforced concrete (red area in Figure 8).



343  
 344 Figure 4 : a) Height of reinforced area, b) Transversal reinforcement, c) Longitudinal reinforcement, d) Vertical  
 345 reinforcement (red areas are homogenized reinforced concrete, blue areas are concrete without any reinforcement)

343 Since the geometry is symmetrical, only a quarter of the beam is meshed. To ensure the induced  
 344 symmetry conditions, axial displacements are blocked on the longitudinal and transversal internal  
 345

346 faces. The displacements are blocked vertically on a line (linear simple support), and a loading line is  
347 also defined for the final loading to failure of the beam (Figure 9).

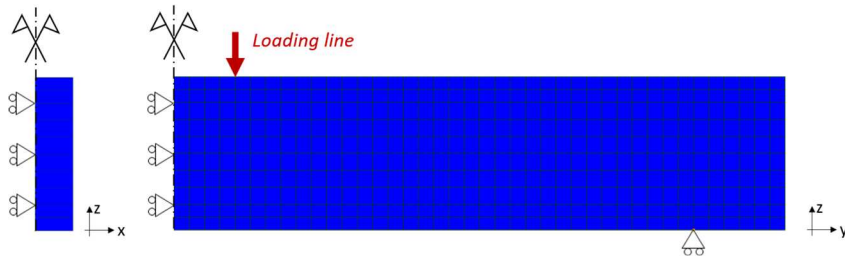


Figure 5 : Boundary conditions

348

## 349 4. Numerical results

### 350 4.1. Ageing phase

#### 351 4.1.1. Structural response of the modelling

352 The ageing period is simulated first. During this phase, the beams were exposed to natural weather  
353 [19]. The seasonal variation of temperature imposed on the beams is given in Figure 10. Note that  
354 these values were provided by the nearest meteorological station. Hence, they may not correspond  
355 exactly to those of the experimental site and, as the region where the beams were stored is relatively  
356 wet, an environment with a constant relative humidity of 90% was assumed for the concrete.

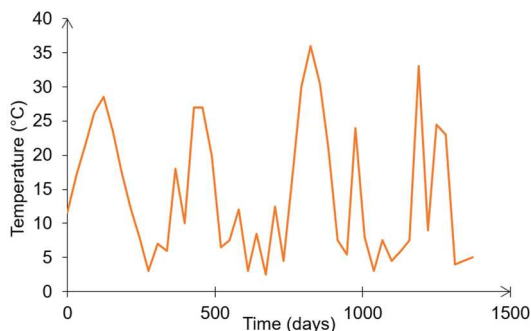


Figure 6 : Set temperature

357

358 Realistic temperatures are used in the numerical simulation to be representative of the real  
359 environmental conditions. Because ASR is activated by the temperature, it is particularly important to  
360 reproduce rapid and slow swelling rates due to low and high temperatures (Figure 10).

361 The vertical and longitudinal strains were monitored on the real reactive and non-reactive beams [19].  
362 To measure the strains, the authors used six monitoring points along the longitudinal direction, and  
363 eight points for the vertical directions, distributed along the whole length. The basis of measurement  
364 was 100 mm and 300 mm for the longitudinal and the vertical directions, respectively. For the  
365 modelling, the points used to evaluate the strains are shown by blue and yellow arrows in Figure 11-a.

366 Although some small variations are visible between numerical and experimental strains, the global  
367 evolutions of longitudinal and vertical strains are well simulated by the model (Figure 11-b and Figure  
368 11-c).

369 The strain of the non-reactive beams is reproduced through a usual dilation coefficient of  $1.10^{-5} \text{ m}^{\circ}\text{C}$ .  
370 The small differences between numerical and experimental data for non-reactive beams can be  
371 explained by the temperature difference between the laboratory and the meteorological station where  
372 temperatures were measured. However, the amplitude of most of the seasonal changes in temperatures  
373 are well reproduced.

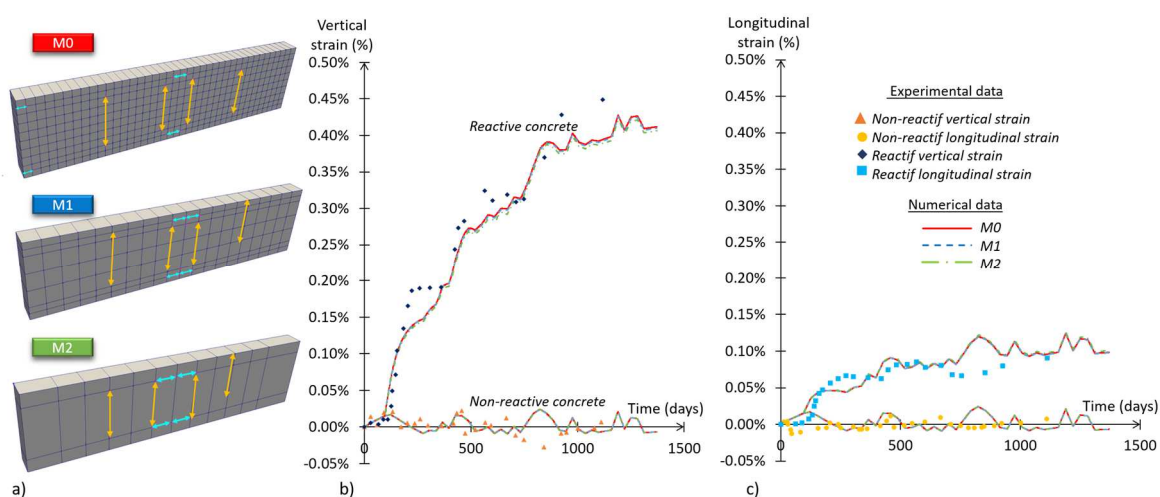


Figure 7 : a) Monitoring points, b) Vertical strain, c) Longitudinal strain

374

375 The anisotropic expansions of the reactive beams are also correctly simulated. They are obtained by  
 376 the parameters given in Table 1. The small longitudinal strain induced by the effect of reinforcement  
 377 on ASR expansion is well-evaluated. It is obtained with the hardening law used to evaluate the  
 378 pressure due to ASR phases in the reactive concrete (Equation (10)). The law was not modified in the  
 379 present work. The calculations used the calibration performed by Morenon et al. in [30] directly,  
 380 throughout the analysis of samples damaged by ASR in laboratory conditions [34]. The interest of the  
 381 law is that it depends on two usual mechanical properties of concrete (the modulus and the tensile  
 382 strength). This leads to robust modelling with no need for new calibration to simulate the behaviour of  
 383 different concretes. In addition, the amplitude of the vertical strain is correctly evaluated and a slight  
 384 overestimation of the longitudinal strain can be observed, thus confirming the capability of the ASR-  
 385 modelling to reproduce anisotropic expansion in field conditions.

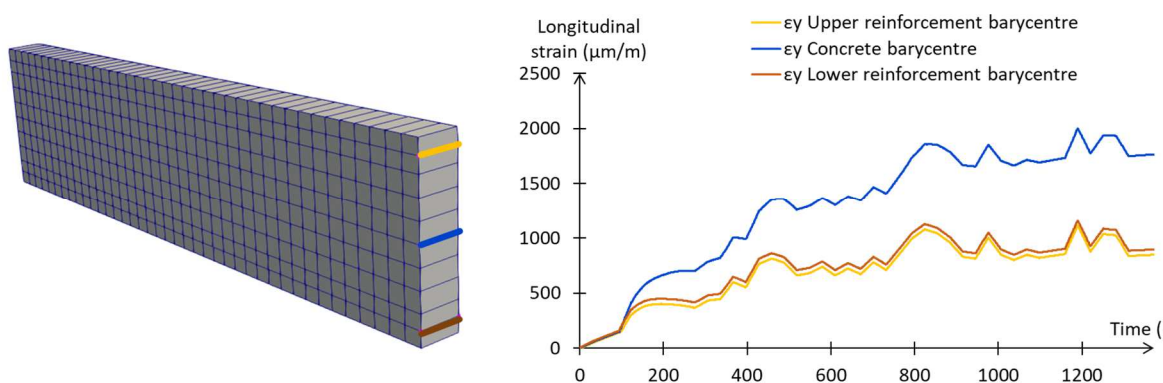


Figure 8 : Strain evolution between the reinforcements and concrete barycentre

386  
 387 To highlight the restraining effect, the differences in longitudinal strains between reinforcement and  
 388 concrete can be observed at their barycentre. Strains are raised on nodes on both sides of the beam,  
 389 averaged according to their height and presented in Figure 12. At the end of the ageing phase and with  
 390 this configuration, the limitation of swelling in the reinforced part is up to 50% compared to the part  
 391 without reinforcement. Besides, due to additional Poisson effect induced by the gravity, the lower  
 392 reinforcement strain is slightly higher than the upper strain (about 50 µm).

Parameter	Symbol	Corresponding	Value
-----------	--------	---------------	-------

		equation	
Total gel production	$\phi^{ASR,\infty}$	(3)	7%
Void accessible by gel under a pressure equal to Rt	$\phi_{ASR}^v$	(5)	1.26%
Characteristic time of the reaction	$\tau_{ref}^{ASR}$	(1)	60 days
Advancement of the reaction at the beginning of observable swelling	$A_{LAT}$	(3)	0.05

393 *Table 1 : ASR input parameter values*

394 The model of homogenized reinforced concrete gives correct anisotropy of the measured expansion.  
 395 The absence of specific meshing of the reinforcement bar does not cause any problem when the global  
 396 deformation of structures damaged by ASR during their service lives is evaluated.

397 At the end of the period, the modelling predicts a mean chemical prestressing of about 2.6 MPa in the  
 398 concrete cross-section of the reactive beams after 17 months and 3.3 MPa after 45 months. This is in  
 399 good agreement with the chemical prestressing evaluated during the experimental programme [19].

400 The model is able to evaluate the diffuse and anisotropic ASR damage (Equation (6)). **The first part of**  
 401 **Figure 13 presents the damage fields according to the three principal directions for the 45 month**  
 402 **reactive case modelled with the finest mesh. Figure 13-b shows a scheme of cracks induced by the**  
 403 **ASR expansion obtained at the end of the ageing period. Diffuse cracking is visible along vertical and**  
 404 **horizontal directions.** As the reinforcement quantity is highest in the longitudinal direction, the  
 405 chemical prestress is greatest in this direction. Hence, the damage is lowest in the longitudinal  
 406 direction and the transfer of expansion induces greater damage in the ~~vertical and transversal~~ **others**  
 407 **directions. This is consistent with the experimental scheme. Nevertheless, the scheme was not**  
 408 **sufficiently precise to obtain quantitative comparison.**

409

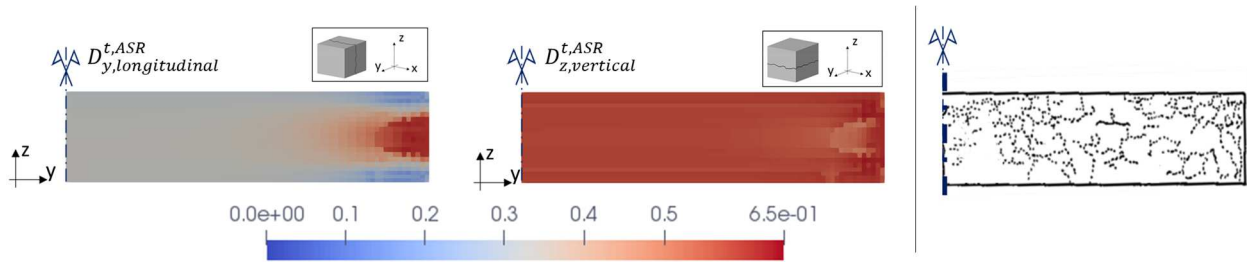


Figure 9 : Anisotropic swelling damage for the 45 month case

410

#### 411 4.1.2. Sensitivity to the mesh

412 Three finenesses of mesh were used. All the results for the ageing period are presented in Figure 11.  
 413 The differences in strain are very small between the solutions obtained for the different mesh densities  
 414 (around  $10^{-6}$  for the non-reactive beam and around  $10^{-5}$  for the reactive one). ASR leads to highly  
 415 intertwined calculations between chemical advancement, damage and creep and hence to numerous  
 416 approximations. It represents relative scattering of less than 1.5% for the longitudinal and vertical  
 417 maximal strain amplitude, and it remains correct whatever the mesh size.

418 The differences in mesh density induce small variations in chemical prestress. Between the finest and  
 419 the coarsest mesh, a maximum scattering of 1.5% is reached. The difference between the finest and the  
 420 intermediate mesh is smaller than 0.5%. For the non-reactive beams, the stress is very small with no  
 421 significant variations according to mesh density.

422 The parametric study highlights the small dependence of the modelling of homogenized reinforced  
 423 concrete on mesh density during ageing due to ASR. Coarse mesh can give a reliable evaluation of the  
 424 service life of reinforced structures affected by ASR and thus reduce computational time.



425 4.2. Bending test to failure

426 4.2.1. Structural analysis

427 4.2.1.1. Load – Deflection curves

428 Following the ageing phase, the bending test is simulated by the application of an imposed  
429 displacement on the loading line. The nodal forces on this line and the vertical displacement at mid-  
430 span are then obtained to establish the load-deflection curves (Figure 14).

431 The load-displacement curves given by the numerical simulations are in accordance with the  
432 experimental curves. The duration of the ageing period has no impact for the modelling of the non-  
433 reactive beams, as hydration is not taken into account (and leads to small differences for the  
434 experimental data). The differences between numerical results and experimental data are greater for  
435 the reactive beams, mainly in the intermediate part, after first concrete cracking and before steel  
436 yielding. In this stage, the model slightly overestimates the force taken up by the beam.

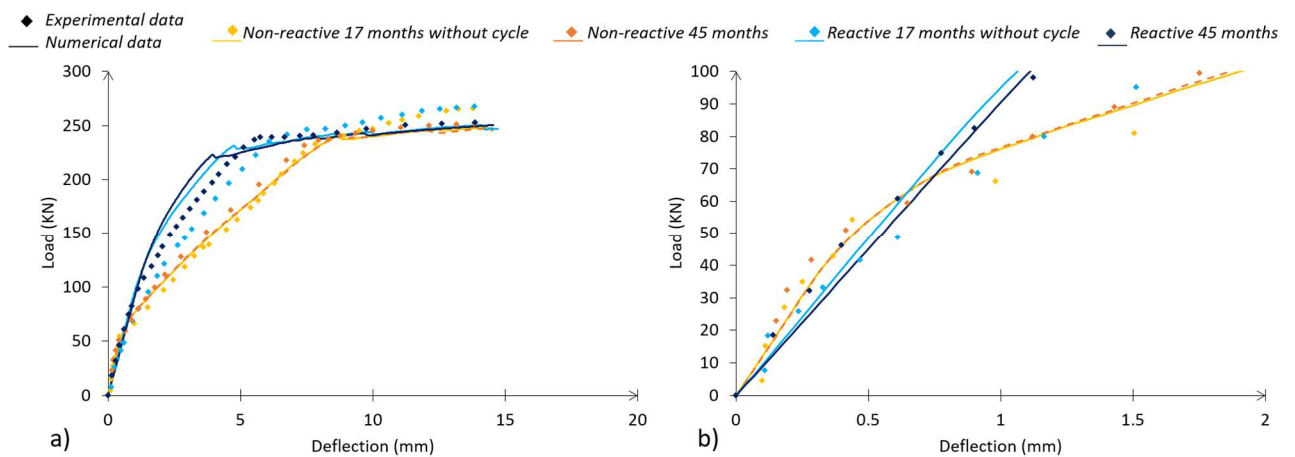


Figure 10: M0 Direct load/deflection curves, a) Full evolution, b) Focus on the beginning

437

438 The differences of behaviour between the reactive and non-reactive beams observed for the  
439 experiments are well reproduced:

- 440 - At the beginning of the loading, the reactive beam is slightly less rigid than the non-reactive  
441 one, due to the damage induced by ASR-expansion during the ageing-period (Figure 14-b)).

- 442 - After the first tensile crack in the lower part of the beam, the trend is inversed. The non-  
443 reactive beams present a rapid decrease of rigidity while the rigidity of the reactive beams  
444 remains unchanged. The conservation of rigidity for the reactive beams is due to the chemical  
445 prestressing induced by the ASR-expansion restrained by the reinforcement bars. It explains  
446 the delay of tensile cracking in the lower part.
- 447 - Steel yielding is obtained for the same loading for all the beams (the simulations reproduce  
448 yielding well). The deflection at yielding is lower for the reactive than for the non-reactive  
449 beams, both in experimental and numerical data. This reduction of elastic domain is due to the  
450 prestressing of reinforcements, which are already under tension before the start of the bending  
451 test for ASR concrete, due to the swelling occurring before the test.

452 In the second part, the impact of the chemical prestressing seems slightly too marked for the reactive  
453 beams. A parametrical study was performed in order to understand this overestimation, and pointed  
454 out the following issues:

- 455 - Evaluation of the rigidity of concrete subjected to ASR: in order to limit the number of  
456 parameters and because of missing data, the model uses some calibrated values (fracture  
457 energy and creep velocity, for instance). In fact, because of their relation to the nature of  
458 the gel and its impact on the matrix, a particular test is needed to establish a quantification  
459 process for some mechanical parameters:
- 460 ○ Damage of reactive concrete is calculated through  $\varepsilon^{k,ASR}$ . This parameter has  
461 already been calibrated on previous works [42]. It drives the impact of ASR  
462 damage according to the value of ASR plastic strain on the concrete strength.  
463 Hence, a mechanical strength test is needed on reactive concrete after the ageing  
464 phase to calculate this value.
  - 465 ○ Evolution of ASR plastic strain is managed by a hardening law, also previously  
466 calibrated [30]. This ratio controls the rise of internal pressure during swelling. To  
467 estimate this law, confining tests could be done during the swelling phase.
- 468 - Availability of ASR-products for new expansion during flexural loading:

469           ○ The assumption on the location of ASR product in porosity has a strong impact on  
470 the response of the model to failure tests. With the assumption that the volume of  
471 ASR products is proportional to the pressure and able to create new expansion in  
472 case of unloading as performed in [7], the rigidity of reinforced beams would be  
473 greatly decreased for the beams studied in this paper (Figure 15) Consequently,  
474 this assumption has been modified: in the present work, it is assumed that ASR-  
475 products are totally blocked in the porosity and cannot lead to new expansion  
476 even in case of a decrease in the compressive stress (see the discussion in part  
477 2.1.3). This new assumption seems to lead to too much rigidity (Figure 15) The  
478 best modelling could be obtained by combining the two mechanisms but, without  
479 clear experimental data or observations on this point, it is difficult to precisely  
480 evaluate the percentage of ASR products able to lead to new expansion during the  
481 flexural test.

482 - ~~Steel-concrete interface Concrete/steel bonding~~ after ASR-expansion in reinforced  
483 concrete:

484           ○ During usual flexural tests on reinforced beams, the loss of bond generally starts  
485 and increases during this intermediate phase of the test. In the present work, the  
486 assumption of perfect bonding between concrete and steel used in the  
487 homogenized reinforced concrete modelling can lead to an overestimation that  
488 could be representative of the real tests. Sliding between concrete and steel could  
489 lead to larger flexural cracks and thus to a greater loss of rigidity than predicted  
490 by the model.

491

492 Future works will focus on this last aspect to improve the precision of the model in the reproducibility  
493 of the behaviour of reinforced structures damaged by ASR after flexural cracking.

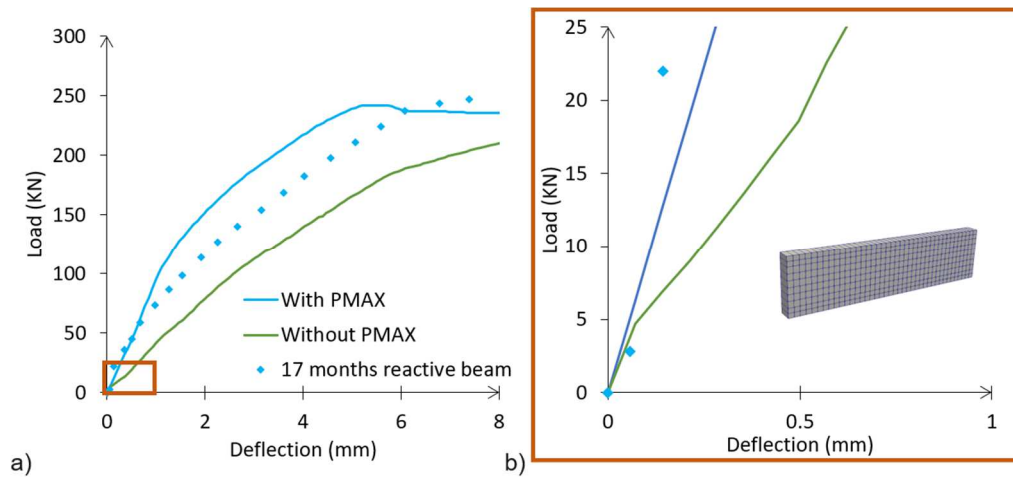


Figure 11 : Evolution induced by the PMAX implementation a)Global evolution b)Focus on the beginning

494

495 Small differences are induced by the ageing phase for the reactive beams from 17 to 45 months. ASR  
 496 advancement is therefore slightly higher in the beam tested at 45 months. It induces greater damage in  
 497 the beam and leads to the difference of rigidity observed in the simulations for loads higher than  
 498 150 KN. Even though the model slightly overestimates the impact of chemical prestress on beam  
 499 rigidity, it is able to recover this ageing difference between reactive beams.

#### 500 4.2.1.2. Load – Deflection curves of cyclic loading

501 Beams tested after 17 months were subjected to a loading cycle before failure. The corresponding load  
 502 displacement curves are presented in Figure 16. The difference between the two behaviours is well  
 503 reproduced and observations are similar to previous results.

504 First, the figure shows the capacity of the damage modelling to reproduce the evolution of the rigidity  
 505 of reinforced structures for usual concrete. Second, despite the slight overestimation of the bending  
 506 force, the structural rigidity of the reactive beam is well-evaluated during the unloading. This confirms  
 507 the correct evaluation of concrete rigidity during this phase. The reason for the overestimation of the  
 508 force during loading has to be sought in other aspects of the model.

509

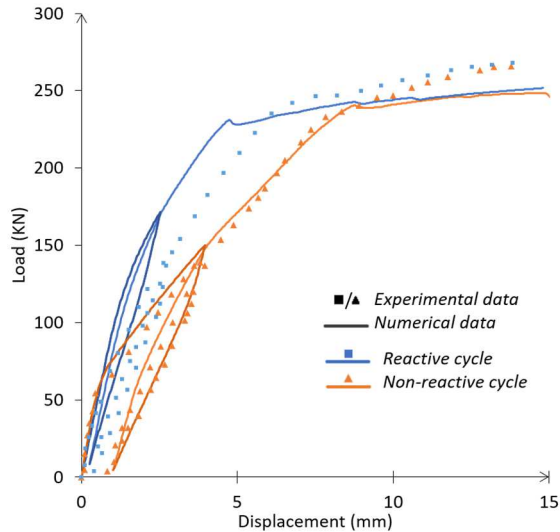


Figure 12 : 17 month load-displacement curves with a cyclic load

510

511 Finally, the present study confirms the conclusions drawn in [19] [4] [50]: ASR has only a slight  
 512 impact on the flexural behaviour of the reinforced beam when beams are calculated for a failure due to  
 513 steel yielding. No significant influence is observed on the ultimate bending behaviour but the  
 514 evolution of the rigidity during the failure test is impacted by the compressive stress induced when the  
 515 expansion is restrained by the reinforcement. Nevertheless, as seen previously in Figure 13, ASR  
 516 induces non-negligible diffuse cracking on reactive beams. Due to these anisotropic micro-cracks,  
 517 reactive beams show poorer durability than non-reactive ones.

#### 518 4.2.2. Structural cracking

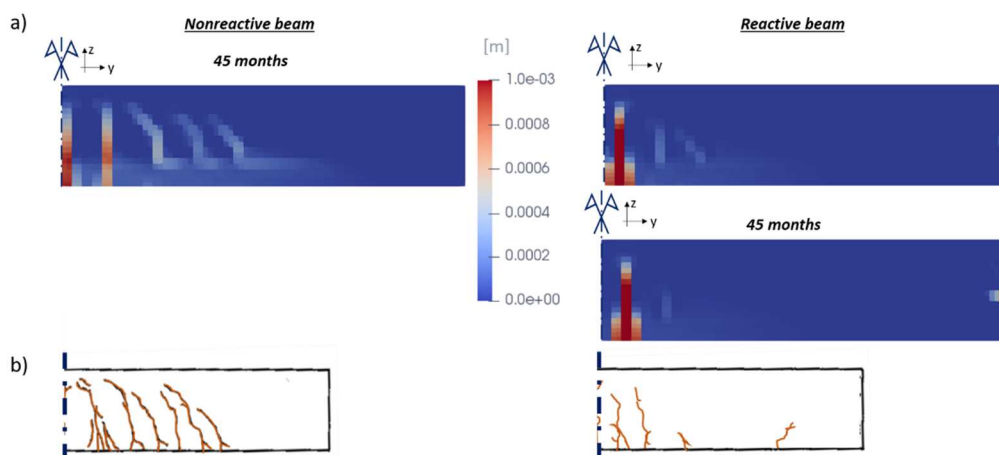
519 The structural cracking observed during the experimental programme is represented at the top of  
 520 Figure 17. The comparison between reactive and non-reactive beams highlights the limited number of  
 521 structural cracks due to the loading of the reactive beam. During experimental observations, about six  
 522 structural cracks were noted on half of the non-reactive beam, versus only two main cracks for the  
 523 reactive beam (top of Figure 17).

524 The structural crack patterns obtained by the modelling are presented at the bottom of Figure 17. The  
 525 meshes show the maximal value of crack opening along the beams. The number, location, and  
 526 direction of cracks are correctly reproduced by the model (with five major cracks for the non-reactive

527 beam and only one for the reactive beam at 45 months). The difference between reactive and non-  
 528 reactive beams in terms of limitation of number of structural cracks for beams with ASR is thus  
 529 reproduced by the model. The difference can be explained by the chemical prestressing of reactive  
 530 beams. The compressive stress induced when the expansion is restrained by the reinforcement  
 531 prevents the opening of the shear cracks close to the supports for the reactive beams. For this reason,  
 532 the non-reactive beam shows almost three shear cracks while the 17 month reactive beam has only two  
 533 thin shear cracks and the 45 month reactive beam has only one, very thin, shear crack. In addition, the  
 534 younger reactive beam shows slightly greater crack opening than the 45 month beam. This difference  
 535 can be explained by the slight difference in chemical prestressing and damage according to the  
 536 duration of the ageing period.

537 The use of homogenized reinforced concrete leads to acceptable reproducibility of the crack pattern at  
 538 the end of a bending test for both non-reactive beams and ASR-damaged beams and it is able to  
 539 reproduce the difference between non-reactive and reactive beams.

540



541

542 *Figure 17 Structural crack opening a)Crack pattern obtained by modelling, b) Scheme of real crack pattern from*  
 543 *[19]*

#### 544 4.2.3. Mesh sensitivity

545 The three densities of mesh are compared in Figure 18. The curves obtained for the non-reactive  
546 beams tested at 17 and 45 months are represented in Figure 18-a and b. The curves obtained for the  
547 ASR-reactive beams tested at 17 and 45 months are represented in Figure 18-c and d respectively.

548 For all the configurations, the flexural behaviour during the failure test shows negligible sensitivity to  
549 mesh size. Even the coarsest mesh - M2 with only 1 finite element in the height to represent the plain  
550 concrete between the two reinforced parts (top and bottom) - gives a correct prediction for the non-  
551 reactive beams. Only the rigidity of the cracked non-reactive beams is slightly underestimated for  
552 loads higher than 100 kN by the two coarsest meshes (M1 and M2). Concerning the curves of the  
553 reactive beam, the variation induced by mesh size is less visible than for the non-reactive beams,  
554 especially between M0 and M1. For both ageing periods, maximal variation is observable in the M2  
555 mesh during the start of the yielding phase. The overestimation of rigidity above the force  
556 corresponding to the concrete cracking for the reactive beams is similar for all the meshes.

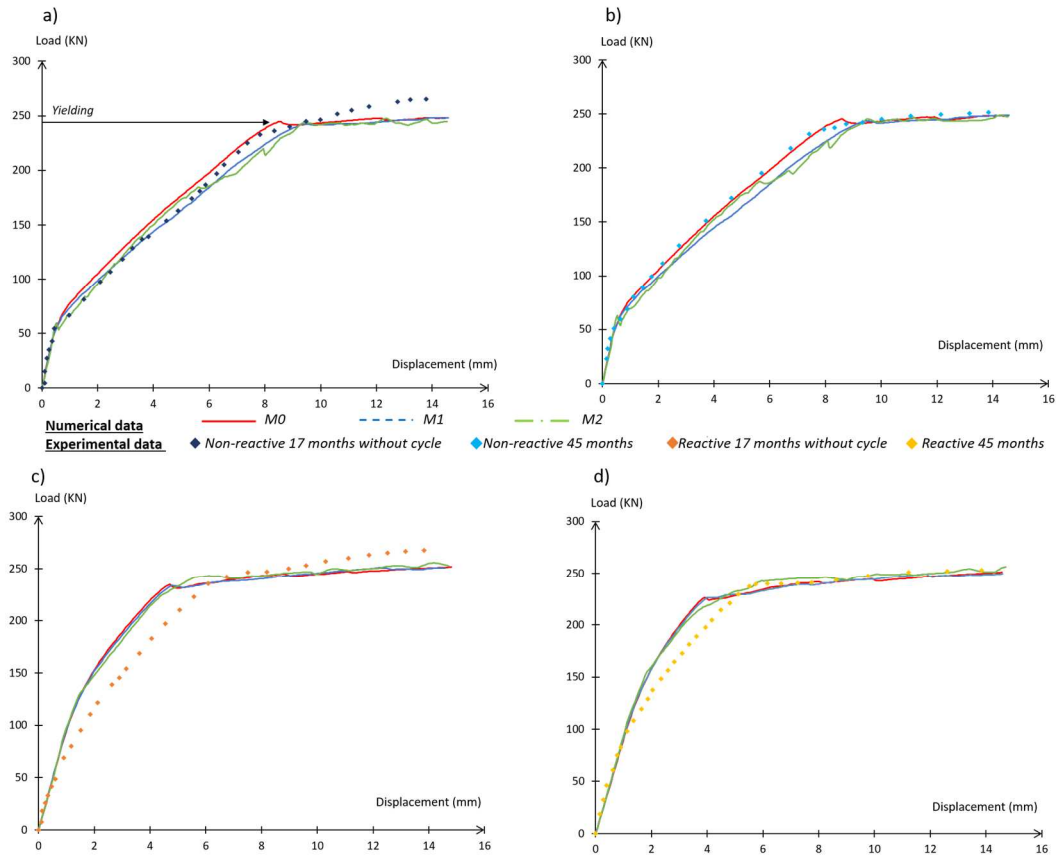


Figure 13 Load-displacement curves a) Non-reactive beam after 17 months without cycle, b) Non-reactive beam after 45 months, c) Reactive beams after 17 months without cycle, d) Reactive beams after 45 months

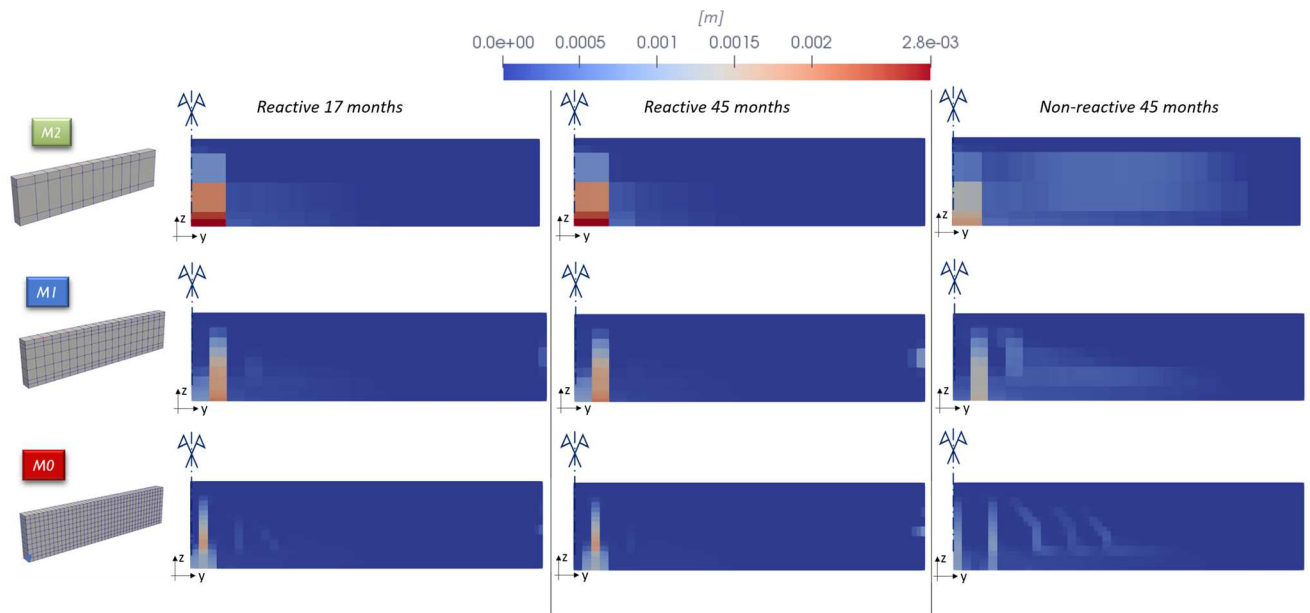
557

558 The low dispersion between these values highlights the independence of the model prediction towards  
 559 the mesh as far as the structural behaviour is concerned, more specifically for the reactive beams.

560 Figure 19 shows the crack patterns obtained for the three meshes. The locations of the structural cracks  
 561 are well-reproduced by the three meshes although there is a loss of crack locations for the mesh M2  
 562 due to the small number of elements in the length. In the non-reactive case, the flexural crack opening  
 563 is almost twice as large with the coarsest mesh (M2) as with the finest mesh (M0), but only one crack  
 564 appears for M2, versus two cracks for the finest mesh. This finally leads to similar results in terms of  
 565 ‘cumulative crack opening’ but with a less accurate location for the coarser mesh. The distance  
 566 between the two cracks observed with the mesh M0 is small compared to the size of the elements used  
 567 in the meshes M1 and M2. These meshes cannot differentiate between two cracks in this zone with the  
 568 element sizes used.



569 The number of cracks in a homogenized element subjected to tension is defined according to the bond  
 570 conditions (anchorage length, bond strength) and the fracture energy [13]. The shear cracks at the end  
 571 of the failure test are well-reproduced for the non-reactive beam with the mesh M0 and, even though  
 572 the modelling is less accurate with the two coarsest meshes, M1 and M2, these cracks are nevertheless  
 573 considered by the tensile damage occurring in this zone.



574 *Figure 14 Structural crack opening for all the meshes*

## 575 5. Conclusion

576 The aim of the present paper was firstly to evaluate the capability of a homogenized reinforced  
 577 concrete model to reproduce the behaviour of beams damaged by ASR during the service life and by a  
 578 failure bending test. Secondly, the stability of the response according to mesh size was estimated.

579 The results obtained for the ageing period show a correct distribution of anisotropic deformations  
 580 with respect to the experimental results. They were obtained thanks to the anisotropic criteria and to  
 581 the hardening law of the model developed in previous work. New calibration was not performed but a  
 582 modification was introduced concerning the nature of the ASR gel, for which a crystallization  
 583 hypothesis was proposed to explain the absence of new cracking during final loading after ageing  
 584 induced by the erasure of the compressive stress present during swelling. Experimental results

585 reproduced in this paper were taken from experimentation performed on ASR beams in natural  
586 weather conditions. The present work shows that the model elaborated from laboratory results is able  
587 to reproduce tests in natural conditions.

588 In this work, the need to explicitly mesh the reinforcement bars is avoided thanks to the use of  
589 homogenized reinforced concrete elements. This is of particular interest for large reinforced concrete  
590 structures, where the meshing of all rebars leads to computational costs incompatible with engineering  
591 processes.

592 The structural behaviours of the beams during the failure tests are correctly evaluated. Simulation  
593 using homogenized reinforced concrete is able to reproduce the load – displacement curves for all the  
594 beams. In particular, it predicts the positive impact of chemical prestressing on the behaviour of the  
595 beams subjected to failure tests, and its evolution over time. It can also reproduce the differences of  
596 structural cracking between reactive and non-reactive beams at the end of the bending test. For the  
597 reactive beams, the chemical prestressing induces a significant limitation of shear cracks, which is  
598 even more noticeable for a longer ageing period.

599 Numerical results obtained with the three meshes present small differences. The impact of element  
600 size on the calculations for this application is negligible but leads to a significant gain in  
601 computational time (which has to be balanced against a loss of precision in the location of cracks).

602 Although the differences between experimental and numerical data are limited, there is an  
603 overestimation on the load/displacement curves of the reactive beams in the intermediate phase after  
604 first concrete cracking and yielding. Various parametric studies have been performed to explain this  
605 fact. The assumption of gel crystallization, formulated to avoid gel returning to the reactive sites  
606 during final loading and causing decompression of the reactive concrete. *While its consideration*  
607 *constitutes an improvement, it also* could explain this slight overestimation of stiffness. An  
608 intermediate assumption could be to consider that some of the gel crystallizes and, in some,  
609 crystallization is delayed. However, this new assumption would require the introduction of a more

610 complex chemical law able to consider ageing of ASR gel induced by an exchange of ions with the  
611 cement matrix. This could be a perspective for the continuation of this work.

612 Despite this slight transient overestimation of stiffness, the homogenized reinforced concrete  
613 approach in an ASR context is able to describe the significant effect of chemical pre-stress on the  
614 behaviour of reinforced structures, without explicit meshing of rebars. Its ability to quantify the crack  
615 opening could be used in the future to assess the evolution of the durability of ASR-damaged  
616 structures and the effects of its combination with other chemical disorders, such as carbonation and  
617 steel corrosion.

## 618 6. Acknowledgments

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