

1 Architecture-Driven Digital Image Correlation Technique 2 (ADDICT) for the measurement of sub-cellular kinematic fields in 3 speckle-free cellular materials

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9 **Abstract**

Measuring displacement and strain fields at low observable scales of complex microstructures still remains a challenge in experimental mechanics often because of the combination of low definition images with poor texture at this scale. This is the case for cellular materials, for which complex local phenomena can occur. The aim of this paper is to design and validate numerically and experimentally a Digital Image Correlation (DIC) technique for the measurement of local displacement fields of samples with complex cellular geometries (*i.e* samples presenting multiple random holes). It consists of a DIC method assisted with a physically sound weak regularization using an elastic B-spline image-based model. This technique introduces a separation of scales above which DIC is dominant and below which it is assisted with image-based modeling. Several *in-silico* experimentations are performed in order to finely analyze the influence of the introduced regularization lengths for different input mechanical behaviors (elastic, elasto-plastic and geometrically non-linear) and in comparison with true error quantification. We show that the method can estimate complex local displacement and strain fields with speckle-free low definition images, even in non-linear regimes such as local buckling or plasticity. Finally, an experimental validation is proposed in 2D-DIC to allow for the comparison of the proposed method on low resolution speckle-free images with a classic DIC on speckled high resolution images.

10 **Keywords:** Elastic Image registration, Finite element DIC, Free-form deformation models,

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12 **1. Introduction**

13 The development of volume imaging opens up attractive horizons in the field of the
14 mechanical characterization of materials, and in particular of architected materials [1]. X-
15 ray tomography, in particular, currently makes it possible to reveal the internal architecture
16 of certain materials at a micrometric scale [2], or even information on the microstructure of
17 metallic materials [3, 4]. The reconstructed volumetric images are therefore commonly used
18 to build so-called Digital Image-Based (DIB) models [5, 6, 7, 8, 9]. Furthermore, by using *in*
19 *situ* testing machines [10], it is possible to assess the effects of loading on internal deformation
20 at various scales [11] or damage [2]. In this context, digital volume correlation (DVC) is now
21 commonly used to obtain a 3D displacement field from a sequence of absorption contrast
22 tomographic images [12]. It is then tempting to take advantage of such measurements to
23 validate the DIB models, or even to identify the parameters of the model used to describe the
24 behaviour of the constituent material(s). However, such comparisons are usually conducted
25 at low spatial resolution and in the case of an elastic behaviour [13]. One of the challenges
26 in the field of experimental mechanics is indeed to perform such DVC measurements at the
27 micro architecture scale [14, 15]. The reason for this is related to the origin of the texture
28 that can be used for image correlation. The typical materials of interest in this study are
29 single-phase materials with complex micro-scale architecture, such as cellular materials. This
30 may include metallic/polymeric foams, biological tissues (trabecular bones), cell woods, or
31 additive manufacturing materials such as lattice structures, to name a few. As an example,
32 an image of a Rohacell-51 polymetacrylimid closed cell foam microstructure obtained using
33 X-ray micro-tomography is given in Fig. 1.

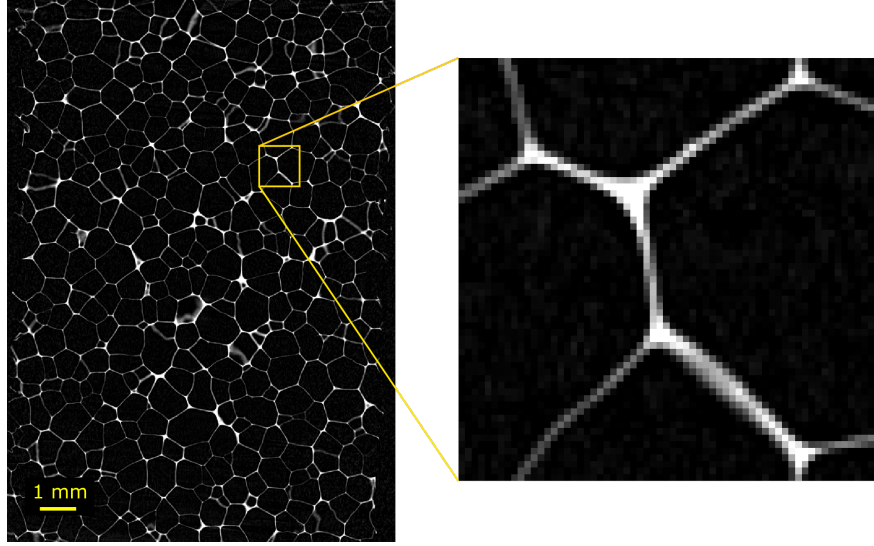


Figure 1: Image of a Rohacell-51 polymetacrylimid closed cell foam microstructure obtained using X-ray micro-tomography. The voxel size is equal to $6\mu\text{m}$ and the cell-struts are defined by only 2 to 3 pixels along the thickness direction.

34 In 2D analysis (DIC or Stereo-DIC), this is possible by artificially adding a high frequency
 35 speckle pattern to the observed surface. Numerous techniques exist that allow textures to
 36 be deposited over a wide range of scales. However, in volume analysis (DVC), depending
 37 on the imaging modality, the variations in grey-levels that generate a DIC suitable texture
 38 are associated with the micro architecture and/or the heterogeneity of the constituents. For
 39 instance in Fig. 1, the acquisition parameters and the size of the sample were such that the
 40 resolution of 6 microns per voxel allows for only 3 voxels on average in the strut thickness.
 41 We can see that the struts are not textured at all. Anyway, with such a resolution, one would
 42 not even be able to see a sub-cellular speckle, even if it existed. With such microstructures,
 43 we are confronted with a paradox: the scale of the constituents is merged with that of the
 44 texture, whereas the texture should be defined at a lower scale. This problem has led DVC
 45 users to consider elements (global DVC) or subsets (local DVC) of very large size compared
 46 to the micro-architecture [12, 16, 17, 18, 19, 20, 15]. The strain fields obtained with such
 47 choices are therefore associated with a meso (or macro) scale which is homogenized with
 48 respect to the material architecture scale. The lack of texture at a smaller scale precludes

49 the consideration of smaller elements or subsets, and therefore to access to more resolved
50 measurements. Of course, there have been attempts to deposit texture in volume, especially
51 in manufactured materials using Barium Sulfate [21] or copper particles [22] as contrast
52 agent for instance. But, apart from the fact that it is not easy to guarantee a homogeneous
53 and isotropic texture and that it cannot be generalised to all materials (especially biological
54 ones), this invasive technique may have effects on the behaviour of the material we want to
55 characterize.

56 This technical barrier which prevents performing strain measurement under the cellular
57 scale represents today's most challenging issue in DVC. For the first time, we propose a
58 method that breaks this barrier and reduces the resolution despite the absence of texture. In
59 order to be able to quantitatively compare the proposed approach (on low resolution images
60 without texture) with a classical method (on high resolution images with painted speckle
61 pattern), we focus in this article on 2D applications. Generalization to 3D, with expected
62 difficulties both in terms of implementation and numerical complexity, will be addressed at
63 a later stage.

64 The method relies on immersed B-spline image-based mechanical modeling for the auto-
65 matic and accurate description of the local kinematic of the imaged sample without using the
66 classical meshing procedures [23]. Then we make use of a tuned equilibrium gap method for
67 the weak regularization of the DIC problem [24, 25]. The 2D numerical and experimental
68 tests are performed on a sample that mimics a slice of a cellular foam as the one of Fig. 1.
69 The novelty of our contribution is a measurement method at the scale of the architecture
70 (using the highest possible spatial measurement resolution) and basing it only on the tex-
71 ture of the sample. As it is based on the use of a regularization model representative of the
72 micro architecture of the material, we called our method Architecture-Driven Digital Image
73 Correlation Technique (ADDICT).

74 As the mechanical response of cellular patterns can be complex and local, the validation of
75 the DIC method must be performed using general mechanical displacement fields that include
76 transformations that are not only reduced to translations and rotations. For this reason, the

77 suggested DIC validation method consists in generating synthetic images of cellular materials
78 from finite element (FE) simulations and comparing the measured displacement fields to the
79 FE reference displacement field. Although it is possible to consider non-linear regularization
80 models [26], the model used here for weak regularization is elastic. The efficiency of the
81 method to estimate local strain fields of samples undergoing possibly non-linear mechanical
82 behaviours is analyzed considering 3 regimes (elasticity, elasto-plasticity and geometric non-
83 linearity) for the generation of the synthetic images. The Tikhonov-like terms used for the
84 regularization of the DIC problem introduces two parameters that are trade-offs between
85 data fidelity and regularity. A detailed investigation of this trade-off is performed based on
86 a L-curve study [27]. Additionally, the influence of the regularization parameters on the
87 true measurement error is performed. Finally, an experimental validation is performed by
88 comparing the results of proposed method on low resolution speckle-free images with those
89 of a classic DIC on speckled high resolution images.

90 The present paper is organized as follows: after this introduction, section 2 reviews the
91 foundations of our approach by recalling the DIC problem and its weak regularization. Af-
92 terwards, we present the automatic image-based model that allows to obtain the geometric
93 and mechanical descriptions of the cell-struts. Section 3 concerns numerical results that are
94 based on DIC virtual tests using an artificial two-dimensional cellular material. In this sec-
95 tion, we firstly compare visually the results of our approach with those of the classical subset
96 method and secondly investigate the influence of the regularization parameters on the mea-
97 sured solution for the three different deformation regimes listed previously. Then, in section
98 4, the proposed DIC measurement method is assessed through a real tensile test. Finally
99 section 5 concludes on this work by summarizing our main contributions and motivating
100 future research based on the proposed methodology.

101 **2. ADDICT: assisting DIC with mechanical image-based modeling**

102 The proposed ADDICT draws on research dealing with FE-DIC [28, 29, 30, 31, 32],
103 weak mechanical regularization [24, 16, 33, 34], and immersed image-based modeling [8, 35,
104 9, 23]. This section introduces the main ingredients of the method and accounts for the

105 choices performed from the current technologies of the literature. More precisely, we start
 106 by outlining the foundations, which are related to an enhanced DIC scheme with weak elastic
 107 regularization, and then briefly describe the constructed specimen specific image-based model
 108 that is the key component of our methodology.

109 *2.1. Foundations: mechanically regularized global DIC*

110 *2.1.1. Global DIC*

111 DIC consists in finding the unknown kinematic transformation that conserves the grey-
 112 level values of the images taken at different loading steps of a material sample. Within
 113 this work, we recall that we restrict ourselves to 2D-DIC but mention that extension to
 114 DVC [12, 17, 18] is straightforward from a methodological point of view. More precisely,
 115 given two images showing two configurations of a material sample (here let us denote f
 116 the image of the material at rest and g the image after load), DIC undertakes to solve the
 117 grey-level conservation equation [36]. Mathematically, it reads: find the 2D displacement
 118 field $u(x, y)$ such that:

$$119 \quad f(x, y) = g((x, y) + u(x, y)), \quad \forall (x, y) \in \Omega, \quad (1)$$

120 where $\Omega \subset \mathbb{R}^2$ is the ROI, and x and y define the coordinates of any point in the ROI.
 121 In practice, the grey-level conservation assumption cannot be guaranteed exactly due to
 122 multiple factors (noise, grey-level quantization, sub-pixel interpolation errors...). Therefore,
 123 problem (1) is rather solved in a least-squares sense for which a distance of dissimilarity is
 124 minimized:

$$125 \quad u^* = \arg \min_{u \in V} S(u) = \arg \min_{u \in V} \frac{1}{2} \int_{\Omega} \left(f(x, y) - g((x, y) + u(x, y)) \right)^2 dx dy. \quad (2)$$

126 In order to do so, images f and g need to be somehow interpolated. In this work, a con-
 127 tinuous B-spline representation [37] will be used, as specified in section 2.2. The unknown
 128 displacement field is searched for in V which is a space spanned by a set of basis functions:

$$129 \quad u(x, y) = \mathbf{N}(x, y)\mathbf{u}, \quad (3)$$

130 where $\mathbf{N}(x, y)$ is the considered shape functions matrix and $\mathbf{u} \in \mathbb{R}^{ndof}$ is the total unknown
 131 degrees of freedom (dof) vector. Depending on the choice made for \mathbf{N} , the DIC methods are
 132 divided into two main families: *subset* methods using mostly low-order piecewise polynomials
 133 that are discontinuous across the subsets [38, 39, 40, 41], and global methods mainly based
 134 on mechanically sound finite elements [28, 29, 42, 43]. Global DIC is considered in this work
 135 since this is the starting point to regularize DIC using a mechanical knowledge of the solution.
 136 In this context, the basis functions defining V can be chosen, for example, as the standard
 137 nodal Lagrange polynomial functions [29, 44, 32], or more regular spline functions in the
 138 spirit of free-form deformation models [45, 46, 47] or isogeometric analysis [30, 31, 48, 23].
 139 In any way, these Galerkin approximations introduce a spatial regularization which is related
 140 to the size and polynomial degree of the considered finite elements.

141 Since problem (2) simply consists in a non-linear least-squares problem, it is solved with
 142 a Gauss-Newton type algorithm [49]. Given an initial displacement guess $\mathbf{u}^{(0)}$, the solution
 143 $\mathbf{u}^{(k)}$ at iteration k is updated as follows:

$$144 \quad \mathbf{u}^{(k+1)} = \mathbf{u}^{(k)} + \mathbf{d}^{(k)} \quad \text{with} \quad \mathbf{H}_S(\mathbf{u}^{(k)})\mathbf{d}^{(k)} = -\nabla S(\mathbf{u}^{(k)}), \quad (4)$$

145 where $\nabla S(\mathbf{u}^{(k)})$ is the gradient of S and $\mathbf{H}_S(\mathbf{u}^{(k)})$ is an approximation using only first-order
 146 partial derivatives of the Hessian matrix of S . These operators are constructed from image
 147 gradients. In the context of the studied images, we perform as usually in the experimental
 148 mechanics community; that is, we actually use a modified Gauss-Newton algorithm which
 149 consists in approximating the terms $\nabla g((x, y) + u(x, y))$ in the Hessian matrix and the right-
 150 hand side by $\nabla f(x, y)$ [49, 50]. This is usually sufficient to capture mechanical kinematic
 151 transformations and has the strong benefit to lead to a constant operator \mathbf{H}_S , which can
 152 thus be inverted once and for all before running the optimization. Further details regarding
 153 the implementation of the method can be found in, *e.g.*, [25, 51, 33].

154 2.1.2. Weak mechanical regularization

155 As mentioned above, discretization (3) introduces a spatial regularization that can be
 156 characterized as a strong regularization in the sense that it is directly related to the size of

157 the approximation subspace. Roughly speaking, to be able to solve the inverse problem (2),
 158 the subset or finite element size must be chosen so that the amount of grey-level data
 159 available in a subset or finite element is richer than the corresponding elementary kinematic
 160 basis. In the conventionally used subset-DIC framework, the usual rule in this respect is
 161 to set a subset size that contains at least 3 speckle dots [52, 53, 54]. For our images of
 162 speckle-free cellular type materials, this would lead to a subset size as depicted in Fig. 2
 163 (see also section 3.4 where further details regarding this image are provided). Obviously, the
 164 resulting approximation space appears too coarse in view of estimating the kinematic fields
 165 at the sub-cellular scale. A finite element mesh as fine as the one of Fig. 2 would be necessary
 166 instead but, in this case, the strong regularization would not be sufficient anymore, thereby
 167 leading to a singular matrix \mathbf{H}_S in (4).

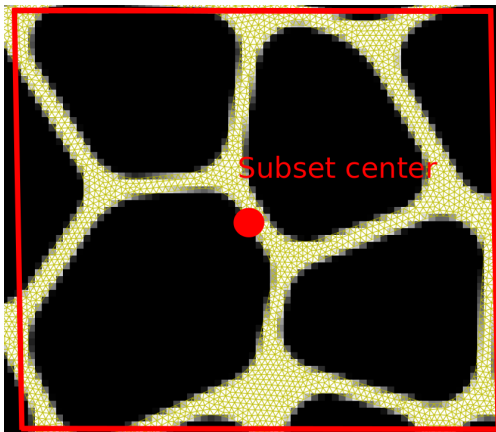


Figure 2: Size of subset (red rectangle) to properly regularize the DIC problem coming from images of speckle-free cellular type materials. The resulting approximation space appears too coarse in view of estimating the kinematic fields at the sub-cellular scale. A finite element mesh as fine as the one depicted in this figure would be necessary instead, thus leading to a severely ill-posed inverse problem.

168 An alternative approach is to resort to Tikhonov regularization techniques [55]. These are
 169 weak regularization schemes that consist in adding to the initial DIC objective function (2)
 170 a specific term, based on differential operators, to smooth the solution fields [56, 57, 20, 58].
 171 In particular, it may be proposed within the FE-DIC technology to penalize the L_2 -norm
 172 of the gradient of each component of the measured field. This technique is often referred

173 to as the Tikhonov regularization technique in the field [59, 60, 19, 48]. In this work, we
 174 will indifferently denote this regularization by the Laplacian-based technique in the sense
 175 that it uses the vector Laplacian operator \mathbf{L} [19, 23], see Eq. (5). More interestingly, using
 176 finite elements in DIC also offers the opportunity to design mechanically sound Tikhonov-
 177 like methods by penalizing the distance between the estimated displacement field and its
 178 projection onto the space of expected mechanical solutions [24, 25, 16, 17, 61, 33, 34]).
 179 This variant will be classified as the mechanically regularized DIC in this paper. In this
 180 work, we use these two regularization schemes together (as in, *e.g.*, [16, 23]): in the part of
 181 the ROI where no relevant physical information is available, we perform a Laplacian-based
 182 regularization, and in the remaining domain where the discrete mechanical equilibrium can
 183 be safely formulated, a mechanically regularized DIC based on an elastic behavior of the
 184 specimen is performed.

185 From a numerical point of view, the Laplacian-based regularization consists in augment-
 186 ing (2) as follows:

$$187 \quad \mathbf{u}^* = \arg \min_{\mathbf{u} \in \mathbb{R}^{ndof}} \left(S(\mathbf{u}) + \frac{\lambda}{2} \|\mathbf{L}\mathbf{u}\|_2^2 \right), \quad (5)$$

188 where λ is the weighting parameter. For the mechanically regularized DIC counterpart,
 189 equation (2) is rather complemented by the L_2 -norm of the internal forces produced by an
 190 elastic model (in the spirit of the equilibrium gap method [62]):

$$191 \quad \mathbf{u}^* = \arg \min_{\mathbf{u} \in \mathbb{R}^{ndof}} \left(S(\mathbf{u}) + \frac{\lambda_K}{2} \|\mathbf{D}_K \mathbf{K}(E = 1, \nu) \mathbf{u}\|_2^2 \right). \quad (6)$$

192 The weighting parameter is this time denoted λ_K . \mathbf{K} is the stiffness matrix of an isotropic and
 193 homogeneous elastic model defined at the sub-cellular scale of the material. The associated
 194 Young's modulus E is fixed to 1 as \mathbf{K} is proportional to E (the influence of E is thus taken
 195 into account through λ_K). \mathbf{D}_K is a boolean dof selection operator that selects the dof located
 196 in the bulk and on the free edges. Such a dof selection appears necessary because we do not
 197 know well the Dirichlet and non-zero Neumann boundary conditions (in practice, we may
 198 barely access to a resultant in one direction). Finally, we combine both schemes (5) and (6)
 199 to regularize each dof of the unknown measured field, which leads to the following enhanced

200 DIC problem:

$$201 \quad \mathbf{u}^* = \arg \min_{\mathbf{u} \in \mathbb{R}^{ndof}} \left(S(\mathbf{u}) + \frac{\lambda_K}{2} \|\mathbf{D}_K \mathbf{K}(E = 1, \nu) \mathbf{u}\|_2^2 + \frac{\lambda_L}{2} \|\mathbf{D}_L \mathbf{L} \mathbf{u}\|_2^2 \right), \quad (7)$$

202 where operator \mathbf{D}_L selects the Dirichlet and non-zero Neumann edges of the ROI, and λ_L is
 203 the weighting parameter for the Laplacian-based part of the regularization.

204 **Remark 1.** *Let us note here that the Dirichlet and non-zero Neumann boundary regular-*
 205 *ization is only used in order to stabilize the measurement at the boundaries. It uses the*
 206 *Laplacian operator so the only physics prescribed on these boundaries is related to a diffu-*
 207 *sion problem. For more mechanically sound regularizations on these boundaries, we refer*
 208 *the reader to other boundary stabilization strategies used in the case of the equilibrium gap*
 209 *method [63, 34].*

210 Finally, it has to be underlined that the (homogeneous and isotropic) elastic behavior
 211 at the sub-cellular scale is not prescribed in a strong way in (7). It is only used as a low
 212 pass filter to alleviate oscillatory effects [16, 17]. From a global point of view, we exploit the
 213 information coming from the movement of cell boundaries (with $S(\mathbf{u})$ in (7)) and weakly
 214 prescribe a locally elastic behavior to softly regularize DIC in the textureless microstructure,
 215 which makes sense in continuum mechanics, even for measuring inelastic fields as will be
 216 demonstrated in sections 3 and 4. In some sense, such a procedure enables to mitigate
 217 the tradeoff between the FE interpolation error (sometimes referred to as model error in
 218 DIC) and so-called ultimate random error (that is related to the ill-posedness of the inverse
 219 problem) [53, 51]. Overall, when using this regularization, three *a priori* input parameters
 220 ($\lambda_K, \lambda_L, \nu$) influence the DIC measurement quality. In theory, a correct estimation of ν must
 221 be provided which remains a problem for this class of methods. However it can be updated
 222 [25]. The problem thus focus on the fine tuning of (λ_K, λ_L) , which will be addressed in
 223 section 3.

224 2.1.3. Functional normalization and physical regularization lengths

225 As the different optimization residuals are not normalized in (7), typical values of λ_L and
 226 λ_K range from 10^1 to 10^9 and their sensitivity to the measured field is not constant across

227 this interval. Besides, the link between λ_L and λ_K and physical lengths is not obvious. As a
 228 remedy, a mechanical interpretation of these regularization schemes has been introduced in
 229 [16, 17]. To start with, a normalization of the residual can be considered using a reference
 230 shear wave displacement v , here chosen in the form:

$$231 \quad v_x(x, y) = \cos\left(\frac{2\pi}{T}y\right), \quad v_y(x, y) = 0, \quad (8)$$

232 where T is the wave-length. The normalization of the functional (7) consists in dividing
 233 each optimization term in (7) by its evaluation at the displacement v . Denoting by \mathbf{v} the dof
 234 vector associated to the finite element discretization of v , the descent direction using this
 235 normalization is therefore given by the following linear system:

$$\left(\mathbf{H}_S + \lambda_K \frac{\mathbf{v}^T \mathbf{H}_S \mathbf{v}}{\|\mathbf{D}_K \mathbf{K} \mathbf{v}\|_2^2} \mathbf{K}^T \mathbf{D}_K \mathbf{K} + \lambda_L \frac{\mathbf{v}^T \mathbf{H}_S \mathbf{v}}{\|\mathbf{D}_L \mathbf{L} \mathbf{v}\|_2^2} \mathbf{L}^T \mathbf{D}_L \mathbf{L} \right) \mathbf{d}^{(k)} =$$

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$$-\nabla S(\mathbf{u}^{(k)}) - \left(\lambda_K \frac{\mathbf{v}^T \mathbf{H}_S \mathbf{v}}{\|\mathbf{D}_K \mathbf{K} \mathbf{v}\|_2^2} \mathbf{K}^T \mathbf{D}_K \mathbf{K} + \lambda_L \frac{\mathbf{v}^T \mathbf{H}_S \mathbf{v}}{\|\mathbf{D}_L \mathbf{L} \mathbf{v}\|_2^2} \mathbf{L}^T \mathbf{D}_L \mathbf{L} \right) \mathbf{u}^{(k)}.$$

237 Let us note at this stage that the left-hand side operator still remains constant and only
 238 the right-hand side is updated during the optimization iterations. Using spectral analysis, it
 239 can be shown that the linear operators \mathbf{L} and \mathbf{K} used for regularization can be interpreted
 240 as low-pass filters (see, again, [16, 17]). More precisely, regularizing using the L_2 -norm of
 241 the second-order differential operators \mathbf{L} and \mathbf{K} can be seen as a fourth-order low-pass filter
 242 acting on the measured displacements on both the bulk and boundary regions. As a result,
 243 the regularization weights λ_L and λ_K can be related to cut-off characteristic lengths denoted
 244 l_K and l_L which verify:

$$245 \quad \lambda_K = \left(\frac{l_K}{T}\right)^4, \quad \lambda_L = \left(\frac{l_L}{T}\right)^4. \quad (10)$$

246 As λ_K and λ_L are dimensionless, the characteristic lengths l_K and l_L have the same unit
 247 as the period T of the shear wave which is in pixels. For a proper study and a mechanical
 248 interpretation of the implemented methodology, the regularization weights will be tuned
 249 in this paper by changing the values of the cut-off wave-lengths l_K and l_L (see section 3 in
 250 particular). The value of parameter T has no real influence on the results: it is just requested

251 to take it large enough so that the wave v can be accurately described by the considered
252 finite element mesh (at least T should be equal to 4 element lengths).

253 *2.2. Specimen specific regularization using an immersed B-spline image-based model*

254 The main feature of our solver (9) is to make use of a stiffness matrix accounting for the
255 cellular architecture to drive DIC within the struts and/or walls of the material. Building
256 such a stiffness matrix requires to investigate the field of image-based modeling which aims
257 at performing mechanical simulation directly on grey-scale data. In this work, we propose to
258 make use of the advanced immersed B-spline image-based model built in [23] which has the
259 interest of being fully automatic, higher accurate and with a proper description of strain fields
260 compared to more standard voxel-based approaches [7, 64], and fairly-priced in the sense that
261 it provides the best possible accuracy (bounded by pixelation errors) while ensuring minimal
262 complexity.

263 *2.2.1. Construction of the automatic and fairly-priced image-based model*

264 We now briefly review the construction of the considered image-based model. Only the
265 fundamentals are given here. For further details, the interested reader is referred to [23]
266 and the works cited hereafter. The model is based on three main ingredients: (i) a level-
267 set characterization of the boundary [9], (ii) a higher-order spline fictitious domain analysis
268 approach, often referred to as the isogeometric Finite Cell Method (FCM) [35] in the field,
269 and (iii) a fine tuning of the related discretization parameters (quadrature rule, element size,
270 polynomial degree) to make it fairly-priced.

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272 More precisely, Fig. 3 summarizes the different steps of the construction of the model.

- 273 • First, a level-set characterization of the material's boundary is performed by construct-
274 ing a binary function that is equal to 1 if the evaluated point is in the region of interest
275 and 0 in void areas (see Fig. 3a). In order to do so, we apply the simple and robust
276 strategy of [9] that consists in building a smooth B-spline representation of the im-
277 age and obtaining a regular contour of the boundary by taking an iso-value of the

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representation.

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- In a second step, the region of interest is embedded in a structured smooth and higher-order B-spline grid for the discretization of the measured displacement field (see Fig. 3b). The matrix \mathbf{N} in (3) contains therefore B-spline basis functions whose supports are dissociated from the actual geometry. This is the key point of fictitious domain techniques that allow for great accuracy and flexibility in image-based modeling. Resorting to smooth B-spline functions is also interesting to properly describe derivative fields such as strains.

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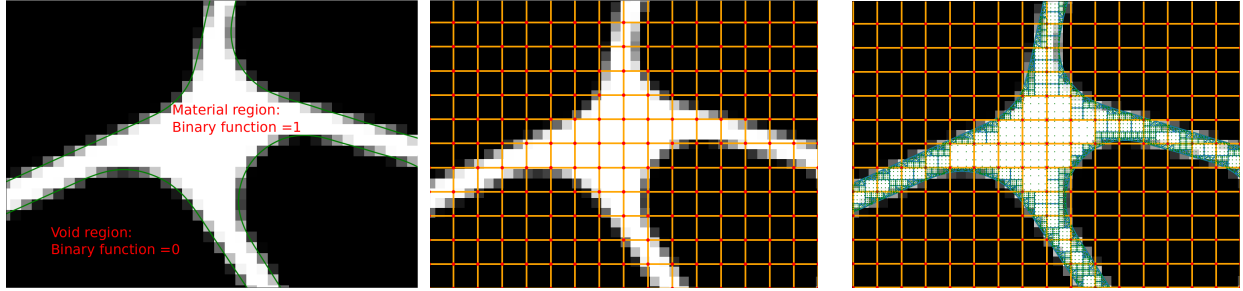
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- In a third step, it is requested to integrate over a restriction of the B-spline grid in order to compute a stiffness matrix related to the physical domain. As the level-set characterization is a signed distance, the integration is performed easily by means of a quad-tree decomposition which is widely used in FCM (see, *e.g.*, [65, 8, 35, 9]). Each element of the B-spline grid is divided into four integration elements if it cuts the boundary (see Fig. 3c). The integration elements that do not cut the geometric boundary are integrated with a full Gauss quadrature. This decomposition is repeated until a predefined maximum level is reached. In addition, in order to improve the geometric description, the last cut integration elements are subdivided into integration triangles equipped with an exact quadrature rule (see Fig. 3c again).



(a) Level-set characterization of the material's boundary.

(b) Embedding of the region of interest in a smooth and higher-order cartesian B-spline grid.

(c) Quad-tree scheme with a closure linear tessellation for defining the domain of integration corresponding to the cell struts (zoomed window *w.r.t.* Figs. 3a and 3b).

Figure 3: Main steps to build the specimen-specific, immersed B-spline image-based model.

296 The three fictitious domain parameters are adjusted following [23]: the maximum level of
 297 quad-tree decomposition is taken so that the minimal size of an integration element is about
 298 the same as the pixel size, and smooth cubic B-spline elements of size approximately equal
 299 to the cell strut thickness are employed. For illustration purpose, the considered cellular-like
 300 specimen is shown in Fig. 4 along with the chosen B-spline mesh that is composed of $n_x = 87$
 301 and $n_y = 64$ elements in the x and y direction, respectively. The corresponding approximate
 302 element size is equal to 2.5 pixels.

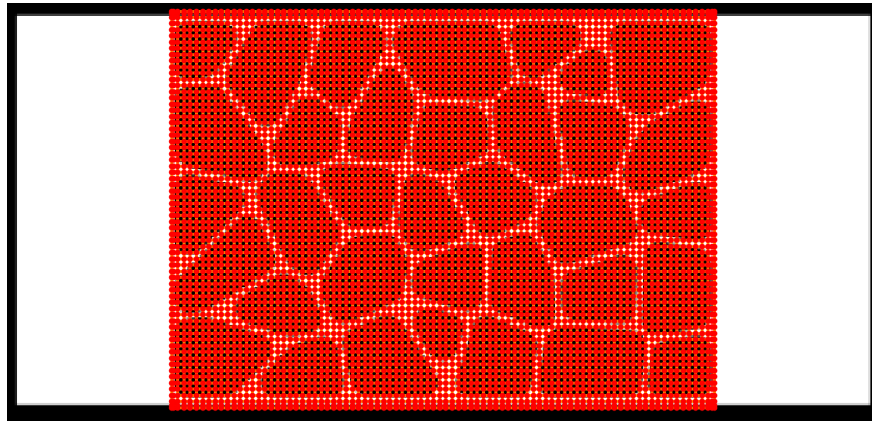


Figure 4: Cubic B-spline grid taken to discretize the measured displacement field for the considered 2D cellular-like specimen.

303 *2.2.2. Conditioning concerns and final fictitious domain DIC approach*

304 In the end, we make use of the B-spline grid and constructed fictitious domain integration
 305 rule not only to compute \mathbf{K} but also \mathbf{H}_S and ∇S (and \mathbf{L}) in (7). In addition, we interpolate
 306 the images by using the smooth B-spline representation constructed at the first step of the
 307 image-based model to define the level-set function, which is interesting from a noise and
 308 gradient computation point of view [66, 9, 67]. The remaining issue to address is that these
 309 operators are in general severely ill-conditioned due to the fact that some basis functions
 310 can have their support that do not or slightly intersect the physical domain. As a remedy,
 311 we remove the dof corresponding the basis function N_i such that [23]:

$$312 \quad s(i) = \frac{\int_{Supp(N_i) \cap \Omega} N_i(x, y) dx dy}{\int_{Supp(N_i)} N_i(x, y) dx dy} \leq \varepsilon, \quad (s(i) \in [0, 1]), \quad (11)$$

313 where $Supp(N_i)$ stands for the support of the considered basis function. In this work, we fix
 314 $\varepsilon = 10^{-4}$ in order to obtain a good compromise between the conditioning of the left-hand
 315 side operator and the accuracy of the solution. In Fig. 5, we show the retained control
 316 points after applying (11) with the considered geometry and mesh. Overall, the strategy (7)
 317 can be seen as an optimized version, using advanced image-based model techniques, of the
 318 mechanically regularized DIC scheme (see, *e.g.*, [24, 16, 17]).

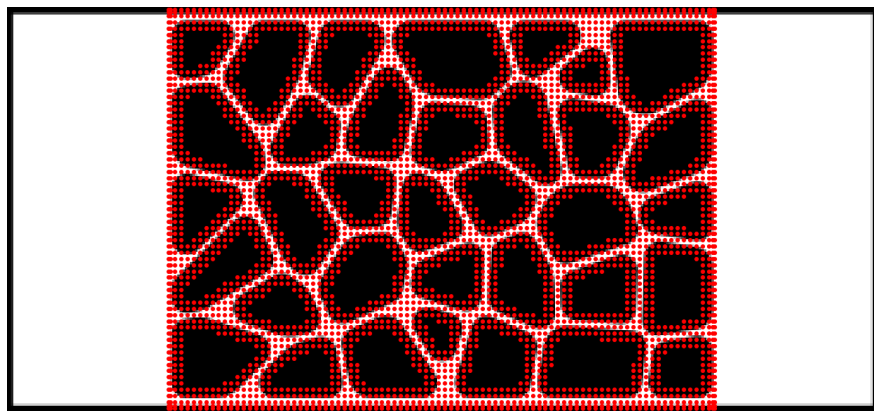


Figure 5: Retained B-spline control points to describe the mechanically regularized DIC solution for the considered 2D cellular-like specimen.

319 **3. Analysis of synthetic images based on virtual tests**

320 In this section, the performance of the proposed speckle-free ADDICT is assessed by
 321 analyzing a set of three synthetic test-cases. Namely, given a fine FE mesh fitting the
 322 architecture of the cellular material, wisely chosen constitutive properties, and boundary
 323 conditions, a displacement field \mathbf{u}^{fem} is computed from a standard FE analysis, as detailed
 324 in section 3.1. Then, synthetic images of the reference and of the deformed configurations are
 325 generated, as described in section 3.2. The interest of such virtual tests lies in the fact that
 326 the measured fields \mathbf{u}^{meas} can be compared with the ground truth \mathbf{u}^{fem} using appropriate
 327 measurement errors, see section 3.3. Fig. 6 summarises the process of constructing and
 328 analyzing images for our virtual experiment. In addition to performing a virtual elastic
 329 test, we will also investigate the ability of our method to estimate local kinematic fields in
 330 non-linear regimes (in particular, plasticity and/or geometric non-linearities).

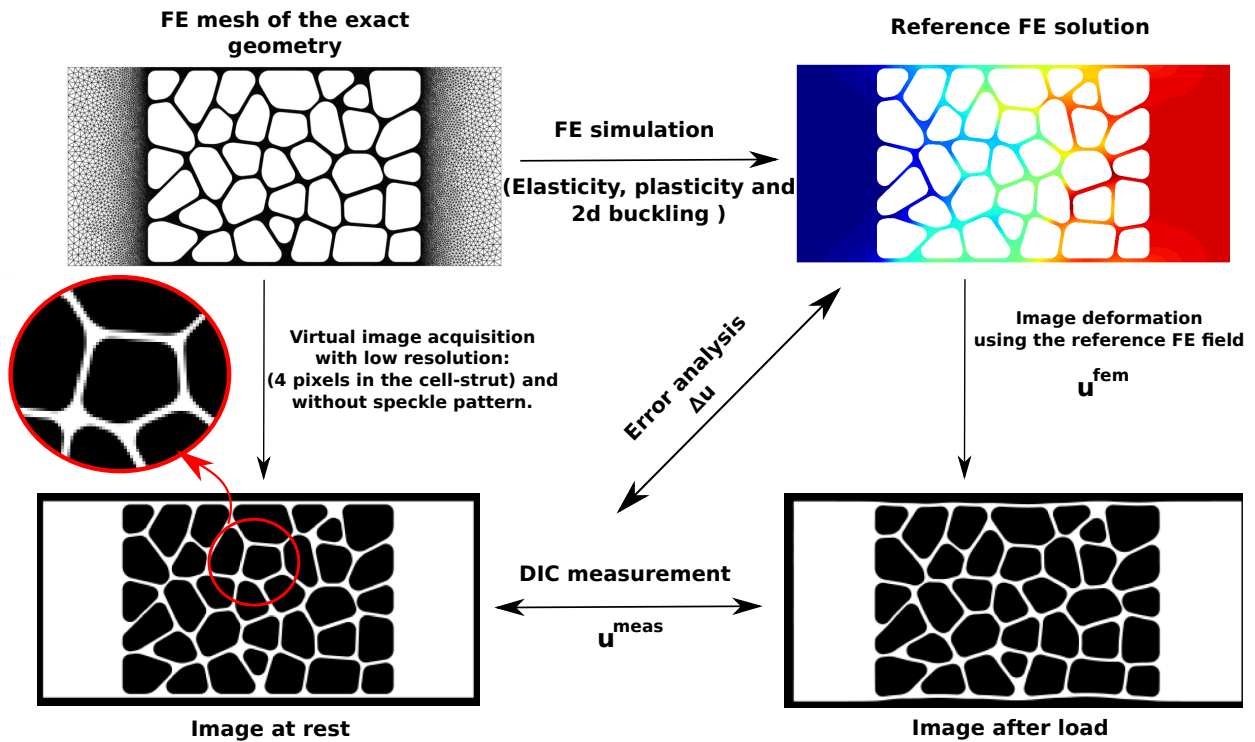


Figure 6: Synthetic image generation and procedure to assess the performance of the DIC measurements.

331 We proceed as follows for the discussion of the results: in section 3.4, it is shown how
 332 challenging it is to estimate sub-cellular kinematic fields with classical subset DIC approaches
 333 from such images. The latter are then analyzed with the proposed method. Finally, for each
 334 of the three test cases, the influence of the regularization cut-off wave-length is analyzed in
 335 section 3.5 based on the so-called L-curves of the optimization problems (5) and (7) and
 336 their relation to the true measurement errors.

337 3.1. Construction of the three virtual tests

338 For the construction of the reference displacement field \mathbf{u}^{fem} , we considered the mechani-
 339 cal problem depicted in Fig. 7. The left boundary of the sample was fixed ($u_x = u_y = 0$) and
 340 an homogeneous displacement was prescribed at the right boundary ($u_y = 0$ and $u_x = u_0$).
 341 The top and bottom boundaries were assumed traction-free ($\sigma.n = 0$). The finite element
 342 mesh was chosen fine enough to correctly represent the local behavior of the cell struts:
 343 approximately six triangular finite elements in a cell strut were considered.

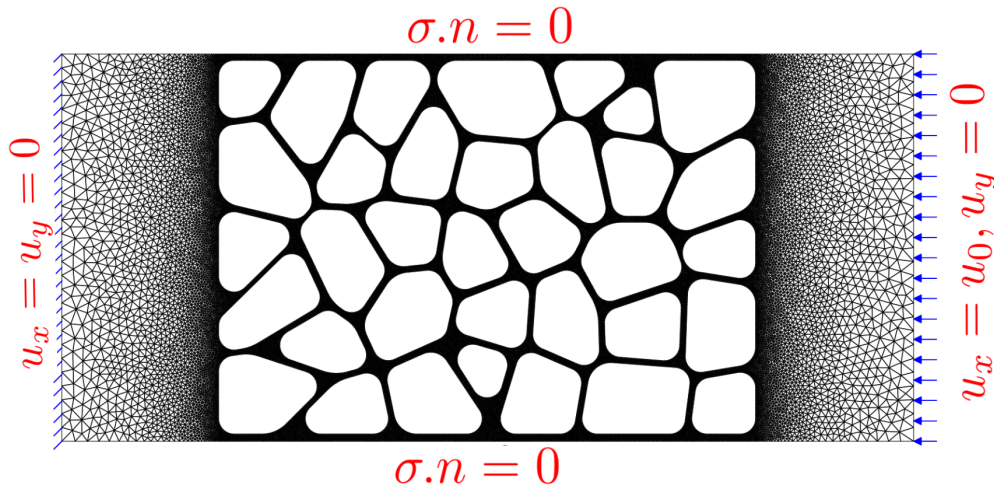


Figure 7: Definition of the virtual experiment: FE mesh of the exact geometric object displayed with the boundary conditions. The sample corners are defined by $x_{\min} = 0$ mm, $x_{\max} = 110$ mm, $y_{\min} = 0$ mm, $y_{\max} = 50$ mm.

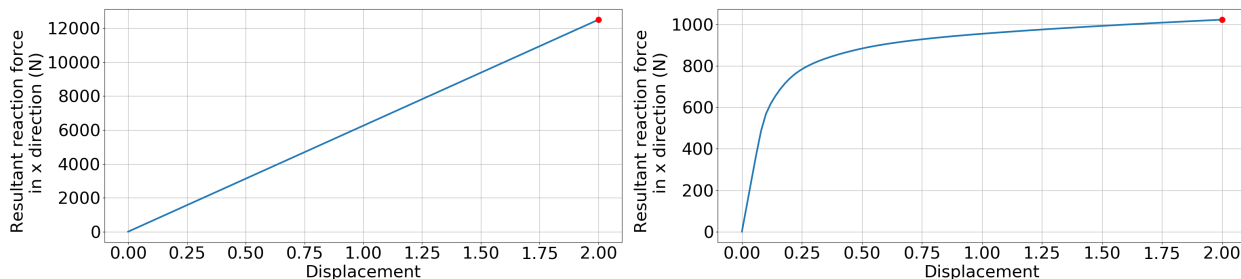
344 In this study, three different mechanical regimes were investigated: (i) linear elasticity
 345 and (ii) non-linear elasto-plastic constitutive relation under infinitesimal strain theory in

346 tension ($u_0 > 0$), and (iii) non-linear elasto-plastic constitutive relation under finite strain
 347 theory in compression ($u_0 < 0$) including post-buckling. For each regime, a Young's modulus
 348 of $E = 187$ GPa and a Poisson coefficient $\nu = 0.3$ were chosen for the sample material. The
 349 material's non-linear behavior was based on the piecewise linear hardening law given in
 350 Table.1.

Plastic strain	0%	0.2%	1%	10%
Yield stress	230 MPa	295 MPa	340 MPa	425 MPa

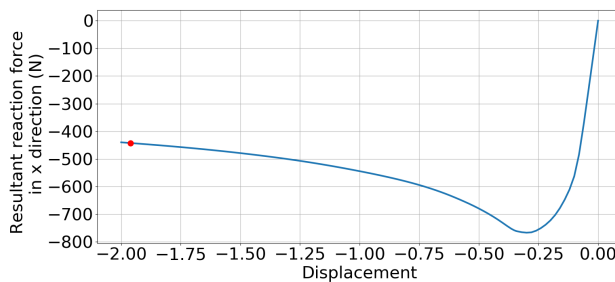
Table 1: Elasto-plastic law used for the reference FE simulation.

351 Figs. 8a-8b-8c show the global force-displacement mechanical response for the three test
 352 cases (i), (ii) and (iii), respectively. The red dots correspond to the mechanical states chosen
 353 to generate the digital images g in the deformed configuration.



(a) Elastic simulation (tensile test $u_0 > 0$).

(b) Elasto-plastic simulation (tensile test $u_0 > 0$).



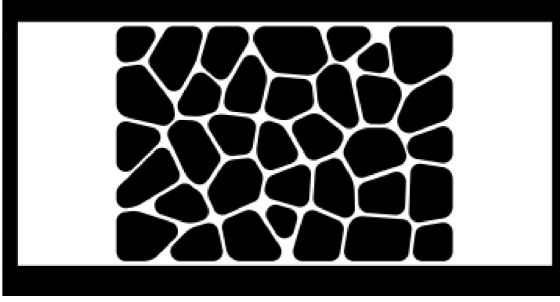
(c) Elasto-plastic simulation with non-linear geometric analysis (compression test $u_0 < 0$).

Figure 8: Evolution of the resultant of reaction forces at the right end of the specimen with respect to the prescribed displacement u_0 in x direction: (a) linear elasticity test (i), (b) elasto-plastic tension test (ii) and (c) geometric non-linear elasto-plastic compression test (iii). The red dots represent the mechanical states used to generate the deformed images.

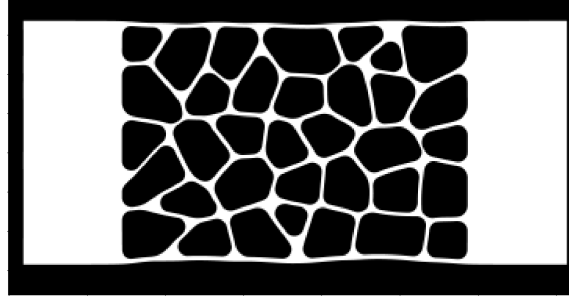
354 *3.2. Generation of the synthetic images*

355 The virtual DIC testing consists in generating a virtual image of the FE model of Fig. 7
356 in the load-free configuration f , and another one after loading g from the above computed
357 displacements fields \mathbf{u}^{fem} . In order to mimic the generation of grey-scale images from the
358 geometry of the sample, a first high-resolution binary image is generated using a cartesian
359 grid of pixels over the rectangle with vertices (x_{\min}, x_{\max}) and (y_{\min}, y_{\max}) . Afterwards,
360 a pixel grey-level value is assigned proportional to its surface fraction to meet the desired
361 low resolution (about 4 pixels in the strut thickness). The same treatment is performed in
362 order to generate the image of the sample in the reference and deformed configurations. This
363 simple rendering method was sufficient in our 2D-DIC analysis whereas other more complex
364 physically sound rendering models could have also been considered, (see, for instance, [68,
365 51, 69] in the context of Stereo-DIC).

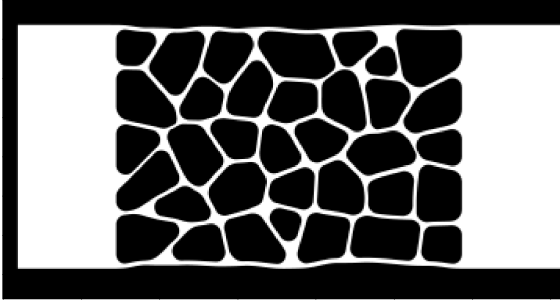
366 Let us recall that the images are chosen for the loading states corresponding to the red
367 bullets in Fig. 8. For the non-linear regimes (see, in particular, Figs. 8b and 8c), this ensures
368 that the behaviour has clearly entered a non-linear regime. The corresponding images f and
369 g are shown in Fig. 9 for each of the three mechanical problems.



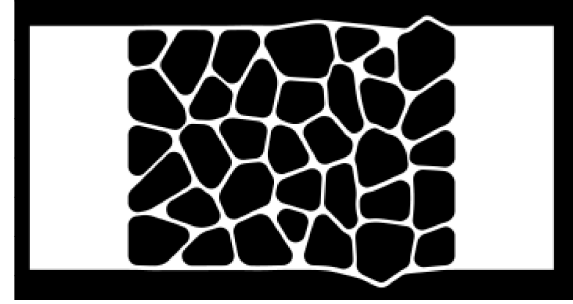
(a) Image of the reference configuration f (load-free).



(b) Image of the deformed configuration for the elastic model subjected to tension corresponding to Fig. 8a.



(c) Image of the deformed configuration for the elasto-plastic model subjected to tension corresponding to Fig. 8b.



(d) Image of the deformed configuration for the geometrically non-linear elasto-plastic model subjected to compression corresponding to Fig. 8c.

Figure 9: Example of pairs of DIC test images based on the same sample but with different mechanical models. Image dynamic is equal to 255 in the whole image area and equal to 127 in the cell area only.

370 3.3. Error quantification

371 As indicated in the overview of the synthetic experimental setup in Fig. 6, the com-
 372 putation of the measurement errors was performed by comparison with the reference FE
 373 displacement \mathbf{u}^{fem} used for generating the synthetic images. Since the reference FE mesh
 374 is consistent with the cell geometry, we choose to compute the error between the measured
 375 $\mathbf{u}_x^{\text{meas}}, \mathbf{u}_y^{\text{meas}}$ and simulated $\mathbf{u}_x^{\text{fem}}, \mathbf{u}_y^{\text{fem}}$ displacements at the n_p Gauss points defined on all
 376 triangular elements of the simulation mesh. In Fig. 10, a zoomed window is provided to see
 377 the FE mesh and corresponding integration points located in the image domain. In order to
 378 quantify the measurement errors, we consider the measurement uncertainty denoted \mathcal{U} . For

379 instance, for the x -component of the displacement it is defined as follows:

$$380 \quad \mathcal{U}(u_x) = \sqrt{\frac{1}{n_p - 1} \sum_{i=1}^{n_p} (\mathbf{u}_x^{\text{fem}}{}_i - \mathbf{u}_x^{\text{meas}}{}_i)^2}, \quad (12)$$

381 where \mathbf{u}_{x_i} stands for the evaluation at the i^{th} Gauss point. The uncertainty \mathcal{U} will be used
 382 for characterizing the measurement error for u_x and u_y with respect to ground truth.

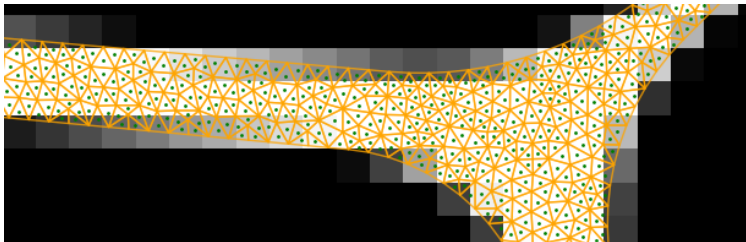


Figure 10: Zoom on an image area. The finite element mesh is superimposed on the image. Green points are the Gauss integration points of the reference triangular FE mesh used for the computation of the error.

383 3.4. A first analysis vs Subset based DIC

384 As mentioned in section 2 and illustrated in Fig. 2, the usual practice in subset based
 385 DIC/DVC is to set a subset size according to the characteristic length of the image pattern.
 386 Based on the auto-correlation function of the image, we can first estimate the microstruc-
 387 ture's characteristic length.

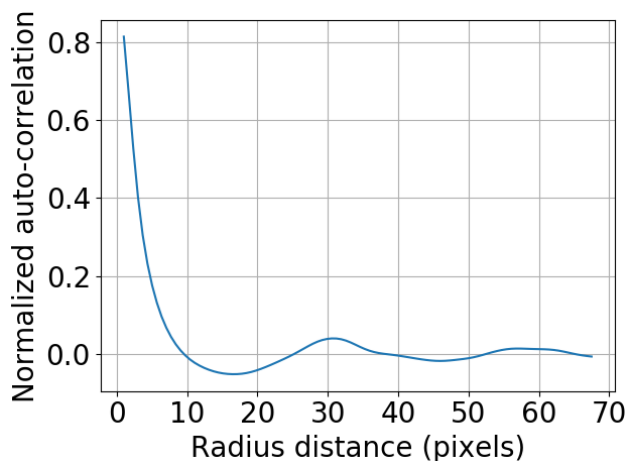


Figure 11: Radially averaged normalized auto-correlation function.

388 More precisely, by performing the analysis of the evolution of the radially averaged nor-
 389 malized auto-correlation, we can estimate an averaged speckle size in the image and the
 390 periods existing thanks to the auto-correlation peaks. The $1/2$ or $1/e$ pre-image of the auto-
 391 correlation can characterize the thickness of a cell strut (here around 4 pixels) [53]. The
 392 secondary peak at around 30 pixels characterizes the mean cell size. Based on the usual
 393 practice in subset DIC [52, 53, 54], it is stated that the subset should contain a minimum of
 394 three DIC pattern features, which leads, in our case, to choose very large subset sizes inca-
 395 pable of reconstructing the local kinematic associated to strut bending (see also discussion
 396 related to Fig. 2).

397 As a concrete example, we consider test case (i) where the underlying model is linear
 398 elastic. The subset-method was applied with affine subset shape functions. In the case of
 399 using the image of Fig. 9a, the subset DIC tool used herein (VIC-2D) suggests an automatic
 400 subset size based on the auto-correlation function. A subset size of 63 pixels is suggested in
 401 this case (approximately 3 pores per subset as shown by the orange square in Fig. 12), which
 402 is consistent with the usual practice. The step size was set to 1. The measurement points
 403 are marked by the red dots in Fig. 12. It should be noted that such a large subset size only
 404 allows measurement in an area relatively far from the edges.

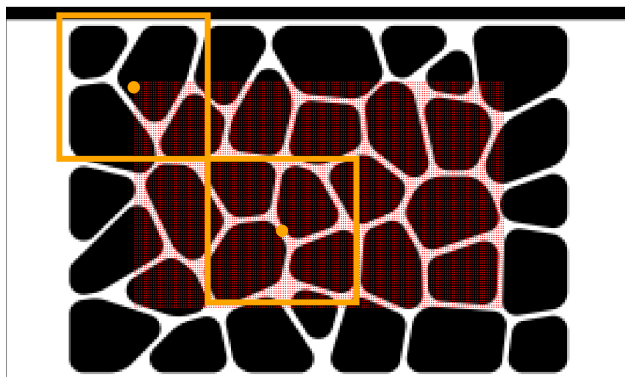


Figure 12: Necessary discretization for the standard subset DIC. The measurement points are marked by the red dots. A large part of the boundary subsets are automatically removed in order to avoid high uncertainty measurements in these zones. The orange square depicts the subset size.

405 A visual comparison of the reference (left) and measured (center) displacements and

406 strains is given in Figs. 13 and 14, respectively. As we are interested by the measurement
 407 within the cell struts only, we show the post-processed results in the cell regions using a
 408 *a posteriori* binary segmentation. In Fig. 13, it can be seen that the displacement field
 409 estimated with the subset method is consistent with the reference field, at least at the
 410 macroscopic scale. But when analyzing the field measured by the subset approach in more
 411 detail, by looking in particular at the strain field in Fig. 14, we notice that the strain provided
 412 by the subset method is completely inconsistent and very far from the reference strain field.
 413 More precisely, the obtained strain fields are homogeneous at the scale of the cell-struts and
 414 the local bending observed in Fig. 14a is not identified. This shows that large subsets only
 415 allow to identify macroscopic (or homogenized) displacements and strain fields.

416 This problem is due to the difficult compromise in choosing the subset size. Indeed, this
 417 parameter alone is used to set both the regularization length and the measurement resolution.
 418 This motivates the use of a richer kinematic (small resolution) associated to an alternative
 419 regularization technique to better capture the sub-cellular displacement field gradients.

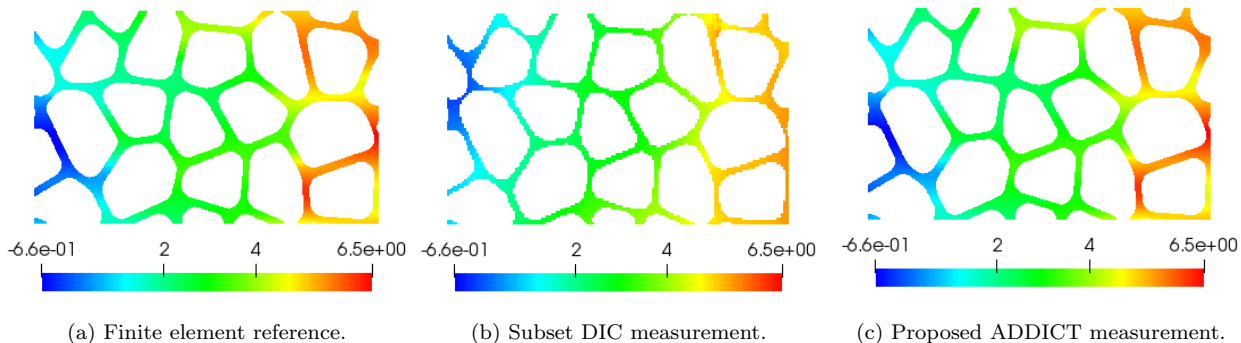


Figure 13: Horizontal component u_x of the displacement field in the ROI of the subset method (in pixel units).

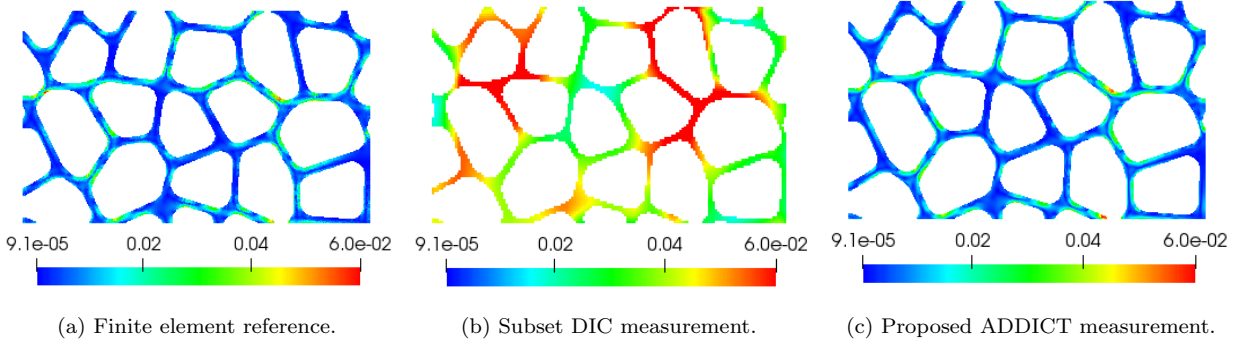


Figure 14: Plot of the equivalent strain field $\varepsilon_{vm} = \sqrt{\varepsilon_{xx}^2 + \varepsilon_{yy}^2 + 2\varepsilon_{xy}^2}$.

420 This same set of images is now analyzed with the proposed ADDICT. An image-based
 421 model, using a B-spline fictitious domain technology, is constructed from the grey-scale
 422 images, as described in section 2.2. This model is used to weakly regularize the FE-DIC
 423 problem, as explained in section 2.1 (see, in particular, Eq. (7)). The corresponding measured
 424 displacement and strain fields are presented in Figs. 13c and 14c. It can be observed that
 425 the displacement field is much better resolved. It shows typical bending gradients which
 426 are quite similar to the reference fields. This is a clear illustration of the interest of the FE
 427 approach in DIC in its ability to use a mechanical model to improve DIC and to break the
 428 aforementioned trade-off.

429 In the following section we will study the two main parameters of our method: (a) the
 430 choice of the regularization lengths l_L and l_K (see Eq. (10)), and (b) the relevance of the
 431 model (here linear elastic) used for the regularization operator with respect to the nature of
 432 the non-linearity of the measured behaviours.

433 3.5. Numerical investigation of the influence of the model and parameters used for the reg- 434 ularization

435 In this section, the influence of the regularization lengths l_L and l_K for different lin-
 436 ear and non-linear mechanical regimes is investigated using L-curves. The L-curve study
 437 of regularized least-squares problems helps finding the optimal regularization parameter as
 438 the one corresponding to the highest curvature point in a log-log plot of the regulariza-
 439 tion term versus the data fidelity term [27]. For our mechanically regularized scheme (see

Eq. (7)), we thus consider on the horizontal axis the dimensionless data-fidelity term defined by $S(\mathbf{u})/(max(f) - min(f))$, and on the vertical axis the variation of the mechanical equilibrium, *i.e.* such that $\|\mathbf{D}_K \mathbf{K} \mathbf{u}^*\|_2^2$. In order to investigate the filtering properties of the equilibrium gap based regularization, the plots are performed for different values of the characteristic lengths: l_L and l_K are respectively varied in $\llbracket 0, 40 \rrbracket$ pixels and $\llbracket 0, 200 \rrbracket$ pixels. The L-curve corresponding to the less physically sound Tikhonov variant (5) is also given for comparison purpose regarding the employed regularization model. In a next step, to account for the relevance of the regularization parameters selected with the L-curve approach, a measurement error study (*w.r.t.* ground truth) is carried out. Eventually, several deformed configurations of the material sample are provided with different values of regularization parameters to appreciate visually their influence on the results.

Linear elastic case. First, let us consider the L-curve when regularizing DIC with our approach (7) in case (i), *i.e.* where the synthetic images were generated with a linear elastic model (corresponding to Figs. 8a and 9b). The obtained plot is shown in Fig. 15. The left and right sides of this figure exactly correspond to the same plot, only the colour of the markers changes. On the left, the colour depends on the value of the edge regularization length l_L , and on the right on the bulk elastic regularization length l_K .

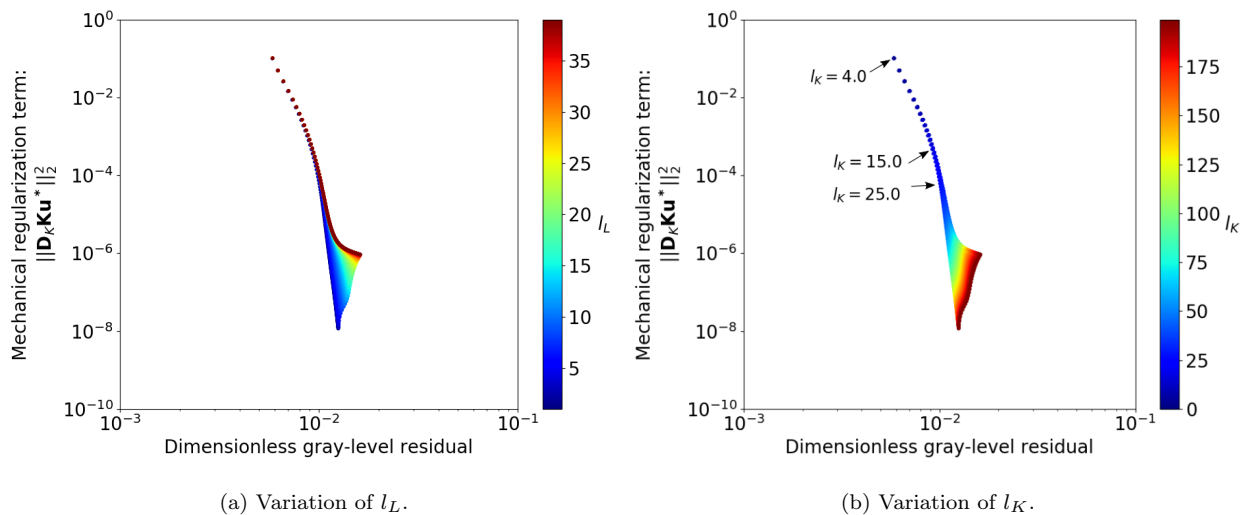


Figure 15: Elastic regularization versus data fidelity for ADDICT on an elastic problem.

457 The first thing that stands out is that the parameter l_L has very little influence on the L-
 458 curve. It only has an effect when the volume elastic regularization parameter l_K is very large
 459 (see bottom zone in the figure), which corresponds to very strong regularization. In such
 460 a situation, it can be seen as an integrated type DIC method [70] which gives good results
 461 provided that (a) the imposed mechanical behaviour in the bulk is the right one (which is
 462 the case on this test) and (b) the edge displacements are relevant. This is the reason why
 463 edge regularization has an effect in this zone. Fig. 15a shows that l_L should be considered
 464 very small (1 to 5 pixels) in order to get an accurate measurement.

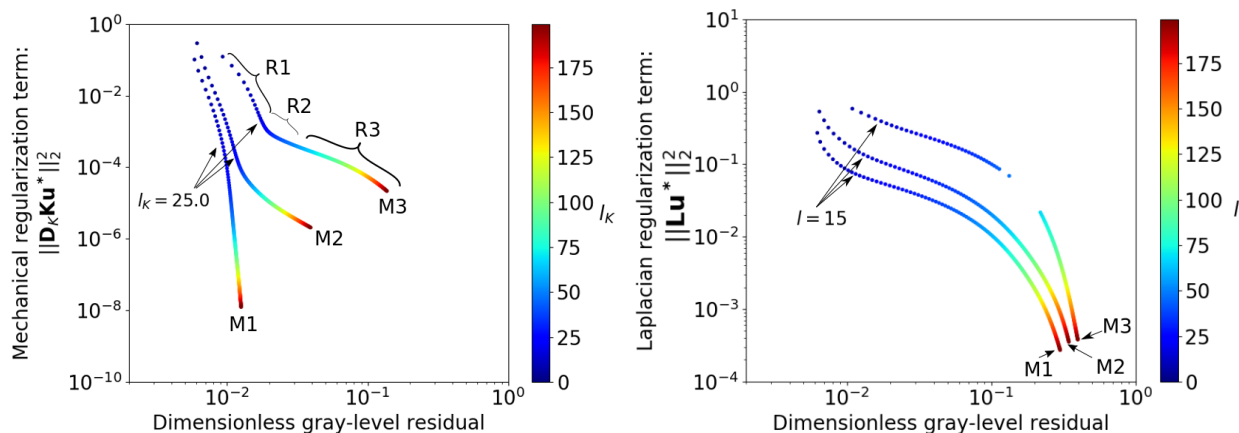
465 Concerning the influence of the bulk regularization given by l_K , while increasing this
 466 regularization weight, the equilibrium term keeps decreasing without a significant increase
 467 of the grey-level residual (the curve somehow plunges down). This implies that the L-curve
 468 does not present a local convexity. The optimal regularization value would be theoretically
 469 infinity. This is the typical behaviour of a perfect (here elastic) regularization term. This
 470 can be observed since the synthetic example actually exhibits a full linear elastic behavior.

471 *Non-linear cases.* The proposed ADDICT with elastic regularization is now applied to the
 472 images of test cases (ii) and (iii), *i.e.* with elasto-plastic constitutive relation without and
 473 with geometric non-linearities, as shown in Figs. 8b-9c, and 8c-9d, respectively. On Fig. 16a,
 474 the corresponding L-curves are presented for the three input models (elastic, elasto-plastic
 475 and elasto-plastic with possible geometric non-linearities). Only the influence of l_K is con-
 476 sidered, l_L being fixed to its optimal value following previous discussion.

477 We can now observe three main regions in the L-curve (denoted R1, R2 and R3 in
 478 Fig. 16a). On the region R1 (*i.e.*, $l_K < 25$), the weight is put more on the grey-level
 479 conservation and the standard deviation is higher, the obtained solution is not accurate as
 480 will be shown in Fig. 17. Conversely, on the region R3 (*i.e.*, $l_K > 30$), the weight is put more
 481 on (elastic) regularity. In this case, the grey-level residual increases as the elastic regularity is
 482 no longer valid for describing the actual mechanics (here plasticity without or with geometric
 483 non-linearities). The choice of l_K must be a compromise between regularity and grey-level
 484 conservation. The optimal value for the regularization length is at the point of maximum

485 curvature [27], i.e. between 25 and 30 pixels, which defined region R2.

486 Through this study, it can also be emphasized that the L-curve is proving to be an
 487 excellent indicator of the relevance of a model in the context of validation [70]. If the
 488 L-curve tends to plunges down as the regularization length increases, then the model is
 489 probably compatible with the observed mechanical field.



(a) Variation of l_K for the proposed ADDICT. l_L is fixed to its optimal value.

(b) Variation of $\lambda = \left(\frac{l}{T}\right)^4$ (see Eq. (5)) for the Laplacian-based regularization.

Figure 16: Influence of the regularization lengths for the three input models. M1: Elastic model (i), M2: Elasto-plastic model (ii), and M3: Geometrically non-linear elasto-plastic model (iii).

490 *Comparison with a less physically sound regularization kernel.* As mentioned above, the
 491 choice of the model used for regularization is one of the two important parameters of the
 492 approach. Here, the less physically sound Laplacian-based model of Eq. (5) was used to
 493 regularize the same set of images. Note that operator \mathbf{L} is built by integrating only on the
 494 physical cell struts (*i.e.* avoiding the holes), which differs from the current practice in other
 495 fields where such regularization operators are used in both strut and void parts [19, 20]. The
 496 corresponding L-curves are given in Fig. 16b. Looking closely at the L-curves of Fig. 16a with
 497 the different regularization operators, we can see that the L-curve is clearly more sensitive
 498 to the increase of the regularization length when using Laplacian-type regularization as
 499 compared to the elastic one.

500 *Link between L-curve and error.* In this section, the L-curves are compared to the true errors
501 in order to numerically validate the optimality of the regularization length associated to the
502 maximum curvature. In Fig. 17, the evolution of the measurement error is plotted as a
503 function of the regularization lengths. We recall that, to compute the measurement error
504 defined by (12), the displacement fields are computed on the Gauss-integration points that
505 belong to both the reference finite element geometry and the constructed geometry using
506 the level-set function. First, this figure provides numerical evidence that the optimal value
507 of the regularization calculated from the maximum curvature point also corresponds to the
508 minimum error. Second, this figure also provide numerical evidence that a weak elastic
509 regularization, even when it is not representative of the actual mechanics of the observed
510 specimen, is better than all the other less physical regularization techniques considered in
511 this study, either in a strong way based on polynomials (subset) or in a weak way based on
512 the gradient of the solution (Laplacian).

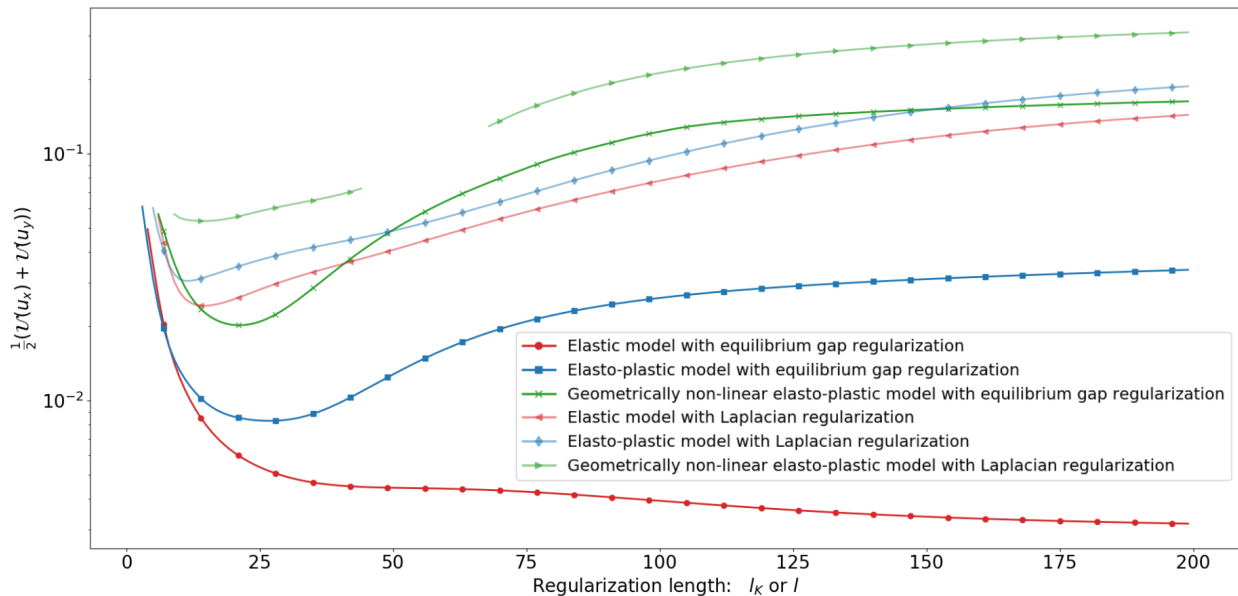


Figure 17: Influence of the regularization parameter on the mean displacement error $(\mathcal{U}(u_x) + \mathcal{U}(u_y))/2$.

513 Overall, the interpretation that can be made of these results is that the term associated
514 with the grey-level residuals ($S(\mathbf{u})$ in (7)) captures the low frequency part of the solution,
515 here associated with characteristic lengths higher than the cell length (≈ 30 pixels), *i.e.* the

516 meso scale. In other words, it helps computing the part of the displacement field that aligns
517 the mesh to the edges of the struts. The local part of the displacements, *i.e.* inside the struts
518 or at the micro-scale, which do not modify the grey-level conservation term, are driven by
519 the regularization. It therefore seems consistent that the optimal regularization length is
520 close to the characteristic cell size.

521 *Deformed configurations with different values of regularization parameters.* In order to visu-
522 ally appreciate the above interpretation, we eventually show several deformed configurations
523 with different regularization weights. First, considering the elasto-plastic case (ii) (Figs.
524 8b-9c), we superpose the reference (red) and measured (green) cloud points for a very low
525 regularization (see Fig. 18a) and for an optimal regularization (see Fig. 18b). Following
526 previous discussion, the low regularization allows to satisfy more data fidelity (region R1)
527 and the optimal regularization corresponds to the inflexion point obtained from the results
528 of Fig. 16a (region R2)). When putting more weight on data-fidelity, Fig. 18a shows that
529 non-physical displacements are observed within the cell-struts as the green points move dif-
530 ferently than the reference points. Conversely, when considering the optimal regularization
531 weight, the movement inside the cell struts is closer to their reference value, see Fig. 18b
532 where the red and green point clouds are superimposed.

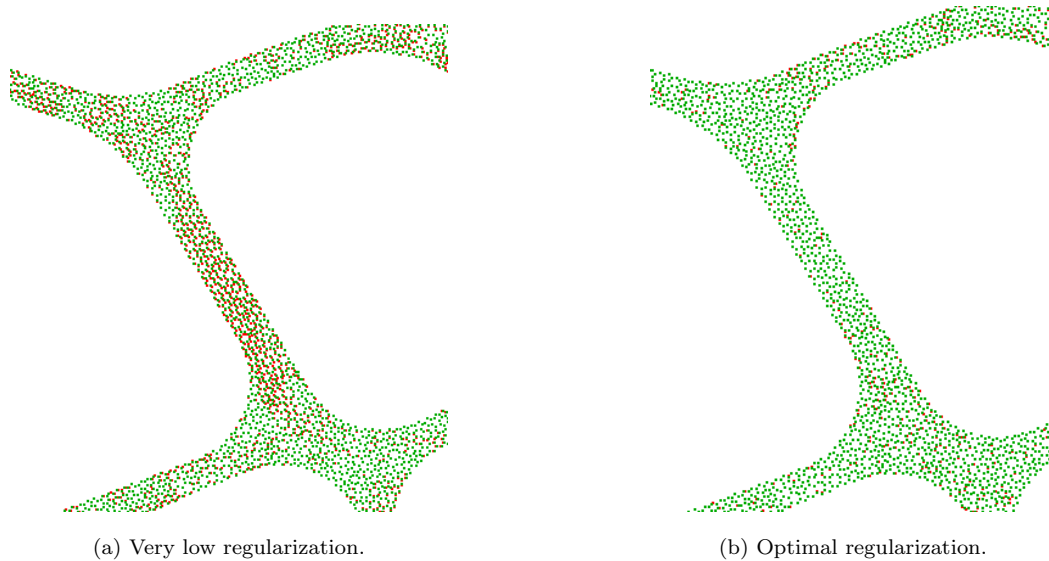
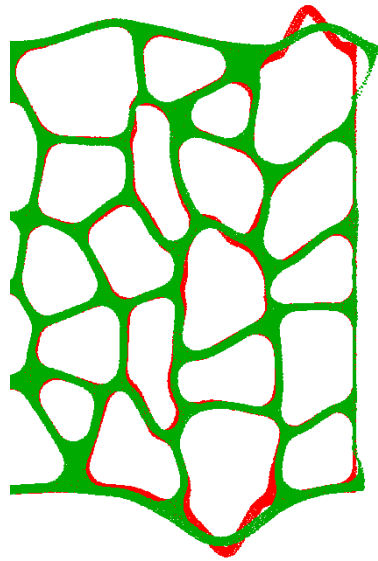
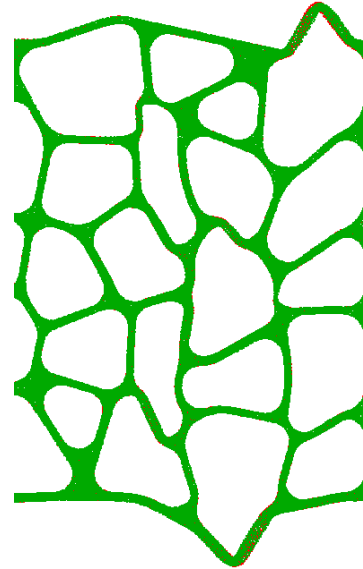


Figure 18: Superposition of the deformed point clouds using the reference finite element field (red point cloud) and the measured field using the equilibrium gap method (green point cloud). Figures corresponding to the elasto-plastic problem (ii).

533 Secondly, in the case of the geometrically non-linear elasto-plastic model (iii) (Figs.8c-9d),
534 when putting a very large weight on the mechanical term (region R3), the correlation fails
535 to correctly represent the geometric non-linearities (see Fig. 19a). In fact, we observe that
536 the regularization model forces the cell struts to bend in an elastic way whereas they should
537 exhibit a post-buckling behavior. When choosing the optimal weight l_K (region R2), the
538 buckling is correctly measured using the same elastic hypothesis for the regularization model,
539 see Fig. 19b. These examples show that even when the observed fields are the response of a
540 more complex behaviour (here geometrically non-linear with elasto-plasticity) than the model
541 used for regularization (here linear elastic), the displacement fields are correctly estimated.



(a) Very high regularization.



(b) Optimal regularization.

Figure 19: Superposition of the deformed point clouds using the reference finite element field (red point cloud) and the measured field using the equilibrium gap method (green point cloud). Figures are corresponding to the geometrically non-linear elasto-plastic problem (iii). (The point clouds are amplified with amplification factor of 2).

542 Finally, Fig. 20 compares the local distribution of strains in the worst case (geometrically
 543 non-linear with elasto-plasticity). Even if the value of the local strain is not totally correct,
 544 it is much better than with the other regularization technique considered in this study, and
 545 it allows at least the location of high gradient areas.

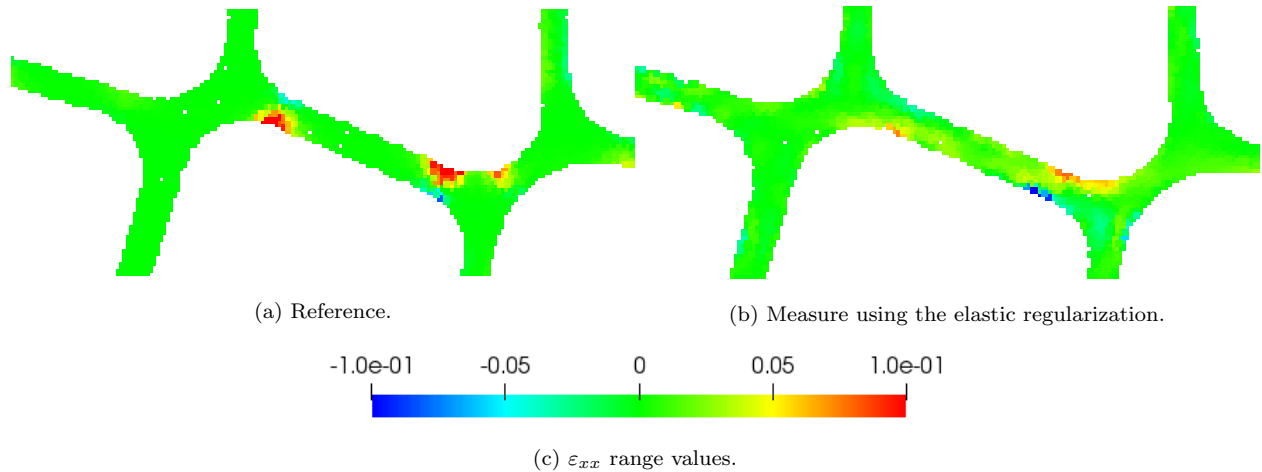
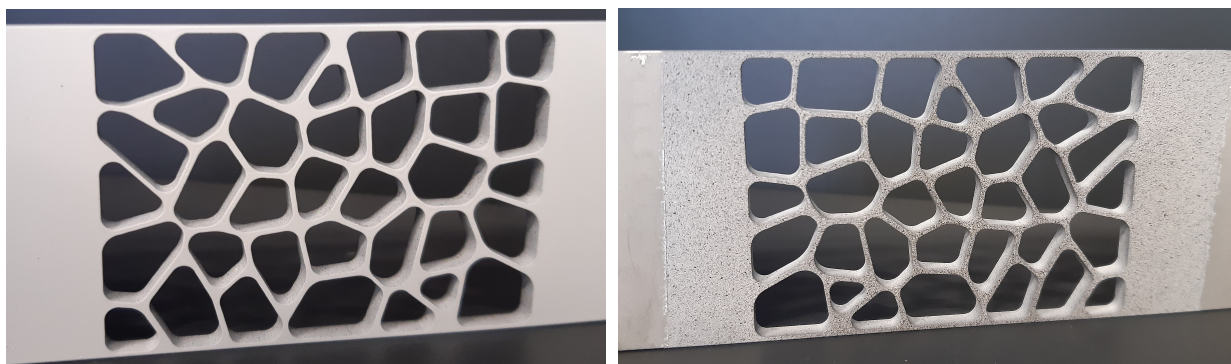


Figure 20: ε_{xx} strain.

546 4. Application to a 2D experiment

547 We now propose to demonstrate the potential of our ADDICT in an experimental situa-
 548 tion where inelastic strains take place. To this end, we have chosen to perform a tensile test
 549 on a macroscopic two-dimensional cellular like specimen and to compare the 2D kinematic
 550 measurements provided by ADDICT using low-definition speckle-free images of the main
 551 side with those obtained by a FE-DIC measurement based on high definition images of the
 552 opposite speckled side, considered as the reference (see Fig. 21). A classic FE-DIC approach
 553 is here preferred for the reference to obtain a dense continuous displacement everywhere in
 554 the struts.

555 We first chose a suitable geometry, material and production method to build our model
 556 material. The geometry adopted is identical to the one used in the previous section (see
 557 Fig. 7). The total width of the specimen is 50 mm, and the minimal struts thickness is
 558 approximately 0.5 mm. The sample was machined in a 4 mm thick 2024-T3 aluminum
 559 sheet from the CAD file using a 5 axis CNC milling machine. This process was preferred to
 560 waterjet and laser cutting in order to obtain the desired geometry while minimizing the heat
 561 affected zone and avoiding the need to deburr the part. The minimum radii of the fillets
 562 were therefore limited in the CAD by the radius of the cutting tool.



(a) Speckle free side for ADDICT.

(b) Speckled side for reference FE-DIC.

Figure 21: Specimen and preparation for DIC - The 50 mm large sample is milled from a 4 mm 2024-T3 aluminum sheet, then painted white between the regions where it will be fixed in the jaws. One side is simply left as it is, while on the opposite side, a speckle is deposited by means of an airbrush.

563 Once machined, properly prepared and cleaned, the sample was sprayed with white matt
564 paint in its entire central region, up to the areas that were to be clamped (see Fig. 21a).
565 Then, thin matt black spots were sprayed on the side where FE-DIC measurements were
566 planned (see Fig. 21b). The idea being to capture displacement gradients within the struts
567 thickness, the deposit of this speckle is done here with an airbrush. Fig. 23b shows the
568 distribution of the speckles obtained on the cell sample. The average diameter of the spots
569 is estimated to be around 0.1 mm.

570 An Instron 8561 100 kN electromechanical tensile machine equipped with a 10 kN cell
571 was used for this test. This machine can be equipped with hydraulic jaws, which avoids
572 accidental twisting of the sample during clamping. Particular care was taken to align the jaws
573 beforehand. The test was carried out under displacement control at a constant displacement
574 rate of 0.12 mm/min.

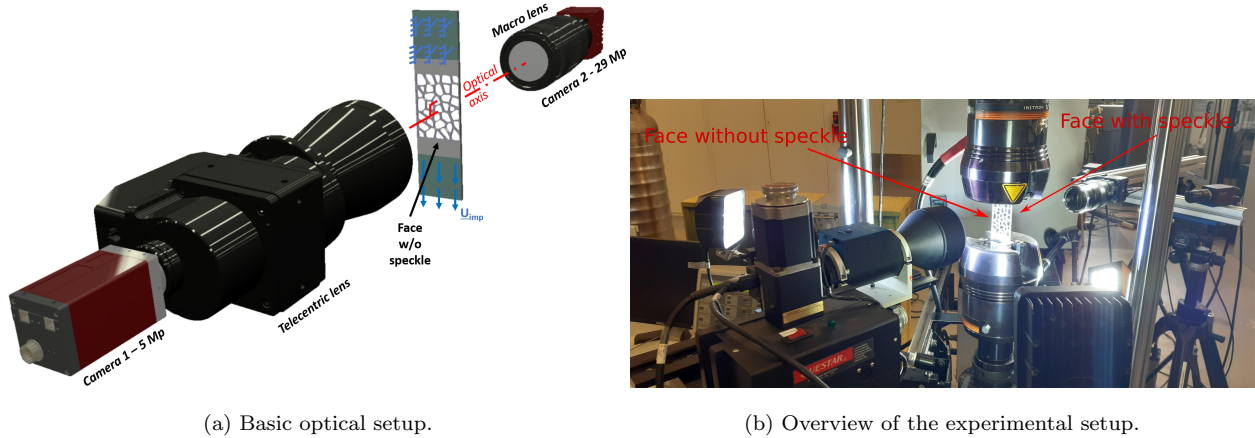
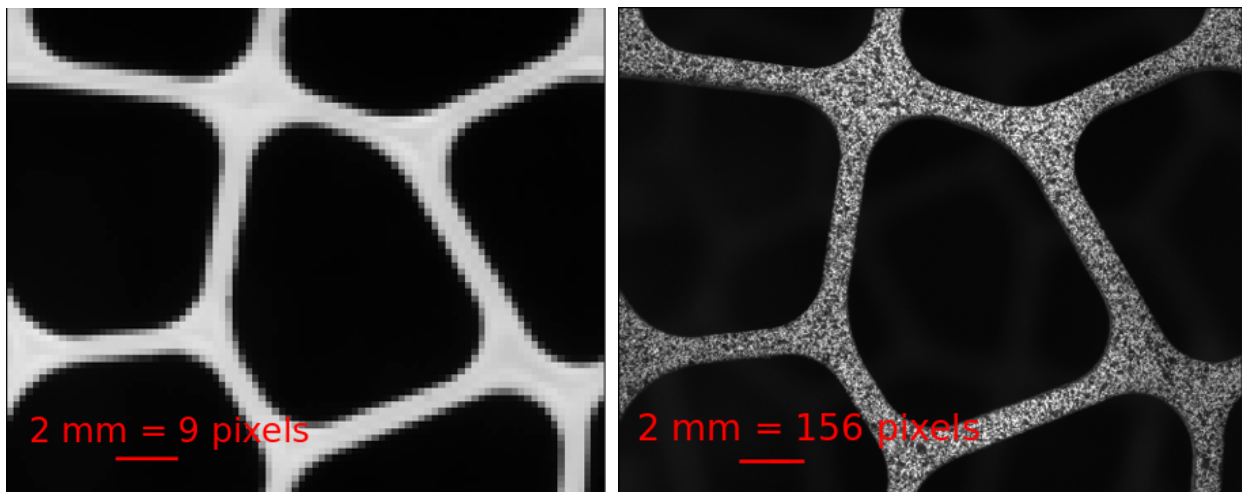


Figure 22: Experimental setup.

575 The experiment was monitored by multiple cameras triggered using an external TTL
 576 square signal. The frame rate was set at 0.2 fps. Fig. 22 shows the basic optical setup
 577 chosen for the present analysis. It consists of 2 systems that were very carefully positioned
 578 on either side of the sample and oriented (using laser devices) so that the optical axes were
 579 perpendicular to the filmed faces. A telecentric lens (Opto Engineering TC ZR 072-C) was
 580 used to film the speckle-free side of the sample. This type of lens allow to maintain the
 581 magnification independently of the working distance and therefore allow to remove depth
 582 effect. It allows here to obtain images of the whole region of interest (field of view: 70.4 mm
 583 \times 52.8 mm). This lens is equipped with a 5Mp CCD camera (Camera 1: Allied Vision Pike).
 584 On the opposite side, a 29Mp CCD camera (Camera 2: Allied Vision Prosilica GT6600)
 585 equipped with a macro lens (ZEISS PLANAR T 2.0/100 ZF MACRO) were rather selected
 586 to retrieve high resolution images of the speckled surface. In this case, the intention was
 587 to correctly resolve the small pattern created on the surface. The working distance of the
 588 macro lens was set to encompass almost the same region of interest (see Fig. 25). The
 589 resulting image has a resolution of about 78 pixels/mm. The zoom presented in Fig. 23b
 590 allows to better apprehend the type of texture which are later treated by the FE-DIC. Note
 591 that the spots are on average more than 7 pixels, which is a little larger than the value
 592 recommended for DIC [54]. The lighting during such an experiment is a problem in itself.

593 It was indeed tricky to light correctly one side without dazzling the cameras placed on the
 594 opposite side. Fig. 22 illustrates how this problem was solved: 2 LED spotlights were used
 595 on each side. This same figure reveals an additional stereo DIC bench in the background.
 596 The latter allowed us to verify that there was no significant out-of-plane movement during
 597 sample clamping or during the test (the maximum out-of-plane displacement measured is at
 598 most a few tenths of a millimeter in the gauge region). This feature will consequently no
 599 longer be used, or commented on, in what follows.



(a) Image of the unspeckled face provided to the ADDICT. Image resolution: 4.5 pixels per mm. Definition of the sub-image presented: 88×73 .
 (b) Image of the speckled face provided to the FE-DIC. Image resolution: 78 pixels per mm. Definition of the sub-image presented: 1218×1558 .

Figure 23: Zoom on a specific region of the sample.

600 The macroscopic load (\bar{F}) - displacement (\bar{U}) curve recorded during the experiment is
 601 plotted in Fig. 24. The dots indicate when the images were captured. For the DIC analysis
 602 which follow, we set the reference image f_i ($i = 1$ unspeckled face, $i = 2$ speckled face) as
 603 the first images captured after the mechanical jaws were clamped (point $(\bar{U}, \bar{F}) = (0, 0)$ of
 604 the curve in Fig. 24). Up to about 3 kN, the sample exhibit an elastic macroscopic response.
 605 Beyond that, the sample undergoes an irreversible strain, highlighted by the discharges.
 606 From now on, we will limit ourselves to present the DIC measurements only for a deformed
 607 state indicated by the red dot on Fig. 24 (point $(\bar{U}, \bar{F}) = (1.05 \text{ mm}, 4.73 \text{ kN})$). The total

608 macroscopic strain is then estimated at 1.5%, while the corresponding residual macroscopic
 609 strain is about 0.8%. The corresponding images are then noted g_i .

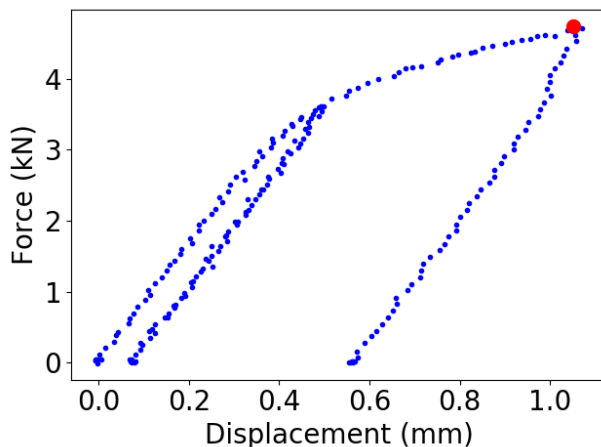
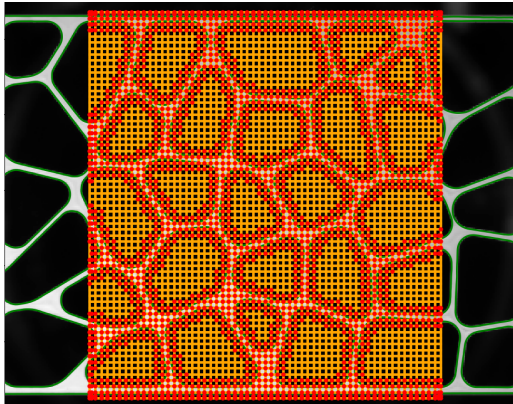


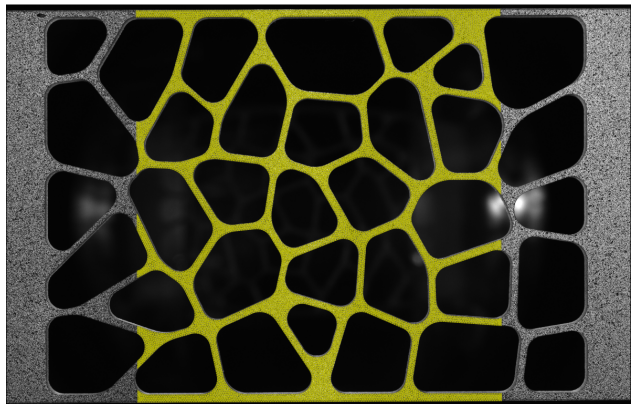
Figure 24: Experimental force (\bar{F})-displacement (\bar{U}) curve. Discharges were performed to highlight the non-linear nature of the deformation. Each point corresponds to the acquisition of images. The red one indicates the state that is analyzed in the sequel.

610 We now propose to measure the displacement fields by image correlation between the
 611 reference state (f) and the deformed state (g) images. The recorded images on the speckle-
 612 free side (f_1 and g_1) are processed by ADDICT. As we want to test our method in conditions
 613 similar to those described above (i.e. with only a few pixels in the strut thickness), the
 614 images are downsampled before being processed. Here, we proceed to three successive data
 615 binning leading to images of 256 pixel \times 306 pixels definition (see Fig. 23a). The resolution
 616 of the resulting images is then about 4.5 pixels/mm. We then automatically define the
 617 implicit geometry of the ROI by building an image-based model as detailed in Section 2.2
 618 (see Fig. 25a). The binary threshold value for the level-set segmentation is here simply set to
 619 $(\max(f_1) + \min(f_1))/2$. Since plastic strains are expected, the regularization parameter λ_K is
 620 set approximately to the optimal value identified in Fig. 17 of section 3.5. When taking into
 621 account the resolution of the experimental images, the corresponding cut-off wave-length is
 622 set $l_K = 50$ pixels. This is confirmed by a new study based on the L-curve. Fig. 26 shows
 623 that the optimal regularization length lies indeed in the interval $[[25, 75]]$ pixels. For their

624 part, the high-resolution images (f_2 and g_2) of the speckled side of the specimen are analyzed
 625 using the open-source FE-DIC library Pyxel [71]. The unstructured T3 measurement mesh
 626 is generated from the very same CAD data used for machining. The average element size
 627 is set to 0.2 mm to ensure theoretically that any element encompasses at least one spot. In
 628 this 2D configuration, the transformation between the mesh reference frame and the image
 629 reference frame (designated projector in this library) is described here with 4 parameters: one
 630 rotation around the optical axis, two in plane translations and one scaling. Those parameters
 631 are automatically identified by imposing that the projection of nodes on the edges must be
 632 aligned with the corresponding edges detected in the images (see Fig. 25b). In practice, we
 633 can check that only a few elements do not benefit from grey-scale gradients (see Fig. 29).



(a) Grid and level-set used to perform ADDICT on the speckle-free face.



(b) FE-DIC mesh used to measure the displacement field on the speckled face.

Figure 25: ADDICT (speckle-free face) and FE-DIC (speckled face) discretizations.

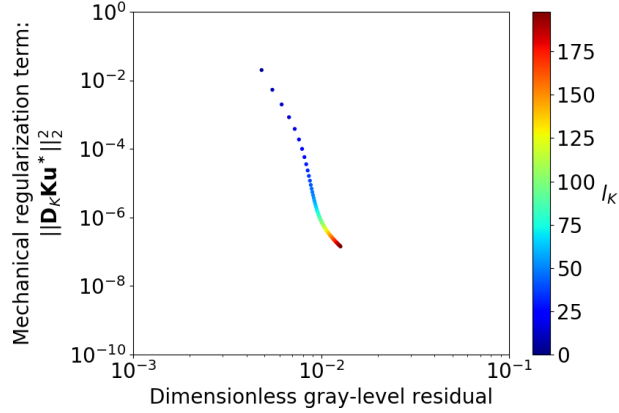


Figure 26: Influence of the regularization lengths for the experimental test-case. Variation of l_K .

634 The longitudinal displacement u_x and transverse displacement u_y fields measured by
 635 ADDICT (exponent 1) and FE-DIC (exponent 2) are respectively compared in Figs. 27 and
 636 28. The maps provided by the two techniques are practically indistinguishable to the naked
 637 eye.

638 A quantitative analysis based on the hypothesis of 2D kinematics is now proposed. In
 639 the present situation, as in section 3, we can indeed directly project the displacement fields
 640 provided by ADDICT on the integration points of the FE-DIC technique (see Fig. 29).
 641 Fig. 30 presents the relative difference between the ADDICT and the FE-DIC measurements
 642 $\frac{|u^1 - u^2|}{\bar{U}}$, where \bar{U} stands for the imposed grips displacement. In no case do the observed
 643 differences exceed 3% of \bar{U} . The local fluctuations for both components are explained by the
 644 uncertainty of the FE-DIC measurement. To complete these comparisons, we propose to look
 645 at the strains inside the struts (see Fig. 31). Not surprisingly, the regularized measurement
 646 leads to less noisy strains and less sharp gradients. Nevertheless, ADDICT allows us to
 647 correctly locate the most severely strained regions. In general, we note that the largest
 648 deviations are observed on the left and right edges of the ROI. This was expected and is due
 649 to the non-physical regularization required on these edges to force ADDICT to converge.
 650 The information provided in the immediate vicinity of these regions should therefore be
 651 taken with caution.

652 In addition to the relevance of the results provided, it should be noted that the use of

653 ADDICT does not require any wizardly parameterisation. Indeed, it should be remembered
 654 that the behaviour chosen for the regularization is elastic, and no optimization of the gray
 655 level threshold to adjust the position of the level-set has been performed (i.e. the description
 656 of the geometry has not be optimized - see Fig. 29).

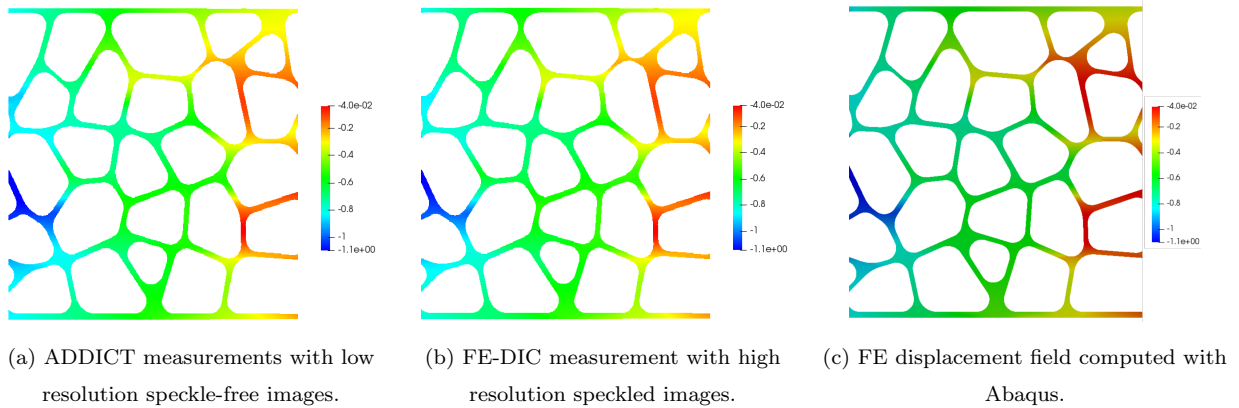


Figure 27: Comparison of the longitudinal displacement fields u_x (mm) measured with ADDICT (u^1), FE-DIC (u^2) and computed with Abaqus (section 3.1) for an imposed displacement $\bar{U} = 1.05$ (Fig. 7).

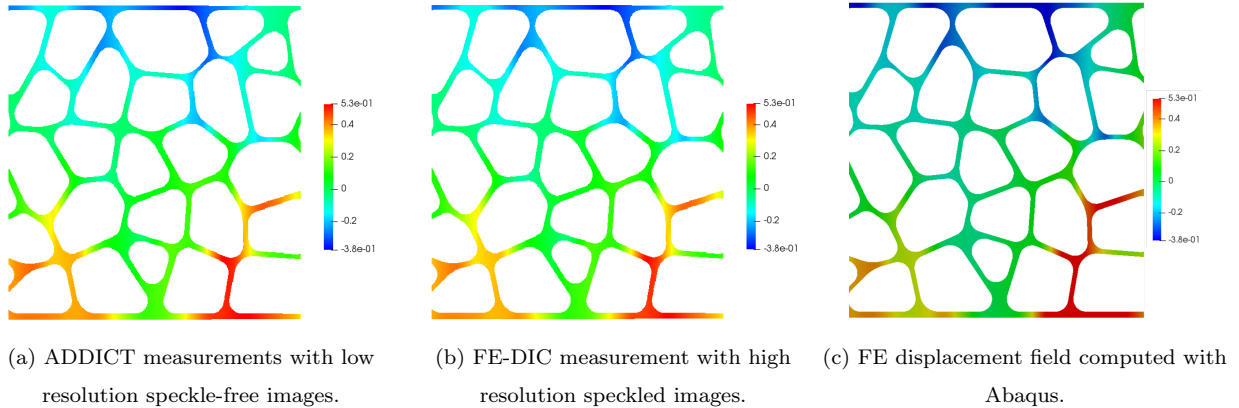


Figure 28: Comparison of the transverse displacement fields u_y (mm) measured by ADDICT (u^1), FE-DIC (u^2) and computed with Abaqus (section 3.1) for an imposed displacement $\bar{U} = 1.05$ (Fig. 7).

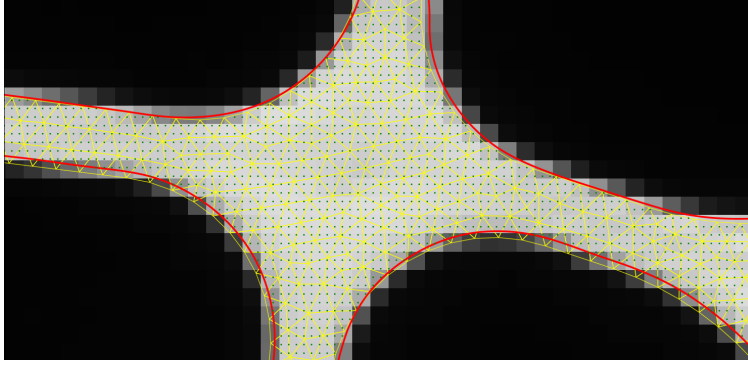


Figure 29: Point cloud belonging to the intersection of the level-set geometry and the FE geometry.

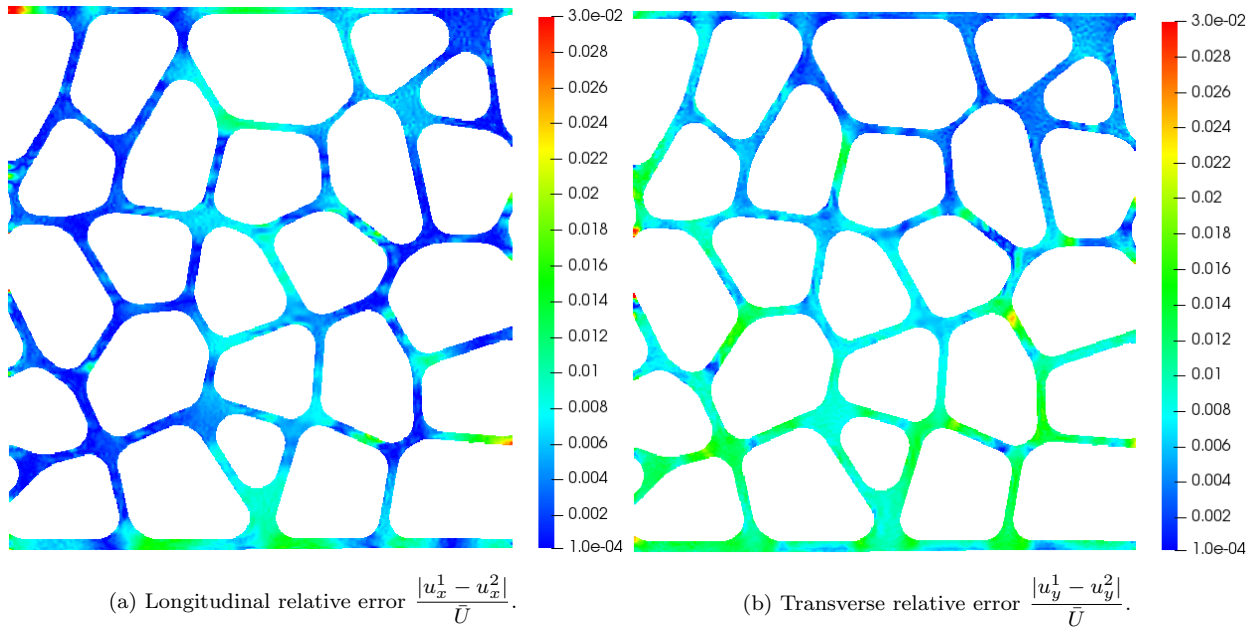


Figure 30: Relative displacement error map between ADDICT (u^1) and FE-DIC measurements (u^2). The difference is scaled by the displacement \bar{U} imposed to the grips.

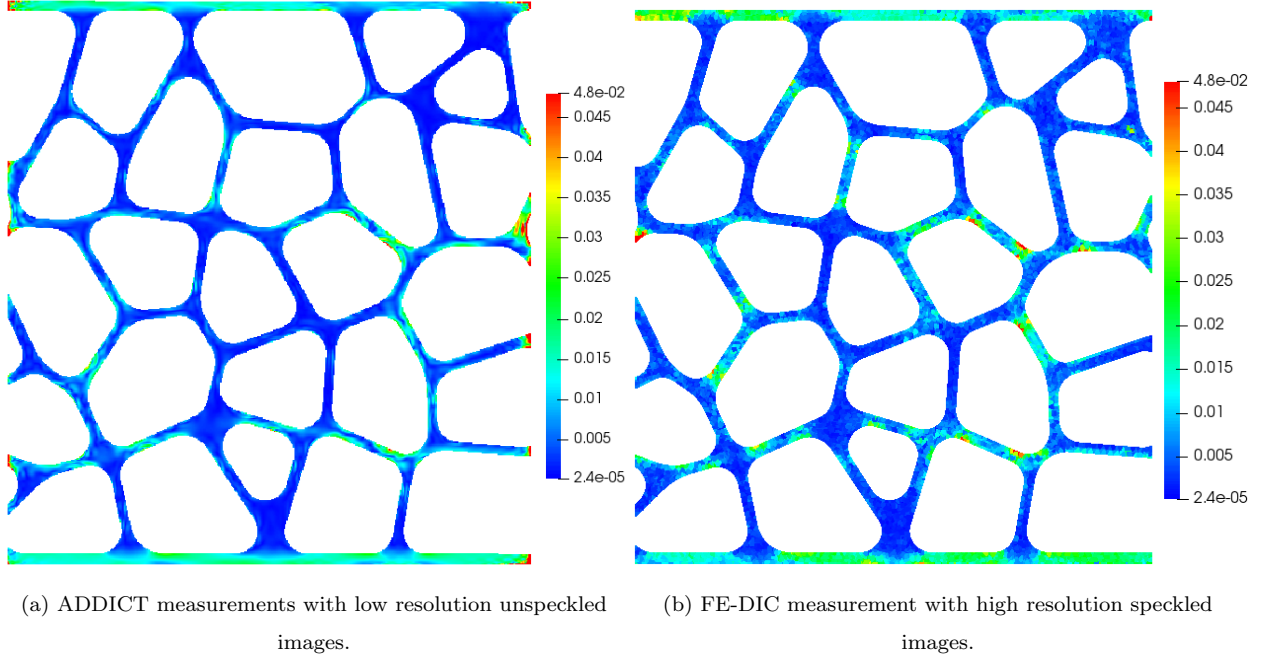


Figure 31: Measured Von Mises strain ε_{vm} .

657 Since the ADDICT measured a relevant displacement field, it becomes possible to validate
 658 a simulation by using only the low resolution speckle-free images. Consider, for example, the
 659 FE model introduced in section 3.1. The constitutive parameters adopted to describe the
 660 elasto-plastic behaviour of the struts are those presented in Table 1. Simple boundary condi-
 661 tions such as those presented in Fig. 7 are adopted. The imposed displacement u_0 is fixed at
 662 the value of the measured grips displacement $u_0 = -\bar{U}$. The longitudinal and transverse dis-
 663 placement fields computed with Abaqus are respectively compared to the measurements in
 664 Fig. 27 and Fig. 28. The observed differences between the simulated and measured fields are
 665 much greater than the difference between the measurement fields. The simulated resultant
 666 \bar{F} is also very different from the load measured at this stage (Fig. 24). This means that there
 667 is clearly room for an improvement of the simulation (ie. discretization, model, constitutive
 668 parameters). Considering that the mesh is sufficiently fine, and that the selected model is
 669 relevant, we could consider identifying the constitutive parameters. A classical FEMU ap-
 670 proach, such as that proposed by [15], but again based on measurements carried out with
 671 speckle-free images, could be adopted. Other identification strategies, entirely in line with

672 the approach initiated here with ADDICT, could also be adopted [25, 26]. Although fas-
673 cinating, this topic is beyond the scope of this presentation and would require a separate
674 study.

675 5. Discussion

676 As stated in the introduction and reported in many papers of the literature, in the absence
677 of texture at a scale smaller than the cell struts, the grey-scale conservation functional alone
678 is unable to estimate local strains even roughly. Nor can it alone identify a strut that localises
679 more strain than others. On the other hand, this functional makes it possible to estimate
680 the macroscopic component of the displacements provided that any sufficient strong (subset
681 or element size) or weak (Tikhonov like) regularization is used.

682 In this study, we showed that it is possible to complement this macroscopic estimate
683 obtained by the grey-level functional with an estimate at the microscopic scale by relying
684 weakly on an *a priori* assumption of the underlying physics. Although not limiting, the
685 assumption used here was linear elasticity, even if the observed behaviour was non-linear.

686 In data assimilation, it is classic to complete a partial measurement with a model. For
687 example, in [43], a stereo measurement is made on the upper (visible) side of a specimen, and
688 the displacements of the lower (non-visible) side are estimated using a model. In a sense, this
689 approach is similar to the one proposed here. More interestingly, the regularization weighting
690 parameter l_K acts as a flexible way to separate the scales: the parts of the displacement of
691 wavelength greater than l_K are handled by the grey-scale metric (if sufficient image gradients)
692 while the ones smaller than l_K by the model.

693 We provided the numerical evidence that (a) the L-curve technique allows to choose this
694 parameter objectively, (b) the optimal length coincides with the minimum of the true error
695 and (c) the optimal length predicted with this technique is fully consistent with the lengths
696 involved in the architecture of the material studied. It is thus not totally indispensable to go
697 through the L-curve study to find a suitable parameter, since observations of the architecture
698 of the material (with possible computation of the auto-correlation) may be sufficient as a
699 first approach.

700 By studying numerous synthetic and real test cases, both linear and non-linear, and with
701 the aim of producing, each time, a reliable reference to compare with, we have been able
702 to show that this method provides reliable local information on the distribution of strains.
703 Indeed, even if the reconstructed geometry does not perfectly match the actual specimen
704 geometry, even if the behaviour is not exactly the good one (elastic vs. nonlinear), we have
705 shown that the method allows to estimate complex local kinematic fields (displacements and
706 strains) in a robust way in very poorly defined images and in the absence of texture. More
707 than that, the method allows to identify the distribution of strains in the various struts and
708 the zones within each strut where the strain localises, despite the poorly adapted input data.

709 An immediate prospect, since ADDICT was built for this purpose, is the extension of
710 this work to DVC to handle real in-situ experiments performed in a μ CT scanner [12, 16,
711 17, 18, 19, 20, 15]. This work is in progress. Such a tool should be undoubtedly valuable for
712 studying the behaviour of a large number of cellular materials (metallic/polymeric foams,
713 bones, wood, additively manufactured lattice structures...). However, the computational
714 cost issue may become a concern in 3D. Domain decomposition techniques or model reduc-
715 tion techniques particularly adapted to the tensor structure of B-splines could then be used
716 advantageously [59, 18, 33]. The DIB model could also be enhanced by other instrumenta-
717 tion modalities (photogrammetry [72], stereo DIC...) A slightly further perspective is the
718 extension of ADDICT to multi-phase materials. Among other perspectives, a very inter-
719 esting avenue concerns the regularization operator. It is indeed possible, with exactly the
720 same formalism, to consider more advanced models (in particular non-linear ones) [26]. In
721 particular, it would be interesting to update the constitutive parameters of the regularization
722 model, which is possible within the very same framework [25, 26].

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