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Highlights

- The influence of flow structures on the design of radiant cooling panel is explored
- Serpentine and Constructal flow layouts are investigated
- A numerical model is employed to evaluate the thermo-fluid performance of the panel
- Branching flow arrangements have the potential of improving the global performances
Constructal design of flow channels for radiant cooling panels

Mohamed Mosa, Matthieu Labat, Sylvie Lorente¹

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Abstract

The main thrust of this study is to bring insight into the influence of the flow channels layouts on the global performance of suspended radiant cooling panels. The flow passages, set on the upper side of a metal plate, represent the crucial part of the panel. Water is conveyed to the panel via these channels to extract the heat flux absorbed by the underneath plate. Therefore, the distribution of the flow over the surface of panel is the key toward efficient panels design. Applying a Constructal approach, the objective of the present work, is to explore the thermal and hydraulic performances of radiant panels equipped with different flow architectures. Including the standard serpentine configuration, branching flow designs are investigated. The flow architectures are categorized into two groups according to the location of the inlet and outlet of the working fluid and further sub-categorized based on the flow arrangements. The influence of the Reynolds number is reported. It is concluded that the proposed Constructal flow structures have the potential of improving the overall efficiency of radiant panels in terms of temperature distribution, cooling capacity, and pumping power demand.

Keywords: Constructal design, Flow architecture, Radiant panel, Cooling, Temperature distribution, Pressure drop.

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## Nomenclature

### Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>area, $m^2$</td>
</tr>
<tr>
<td>$AR$</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat capacity, J/(kg.K)</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter, m</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational constant, $m^2/s$</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer coefficient, W/(m$^2$.K)</td>
</tr>
<tr>
<td>$I$</td>
<td>identity matrix</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity, W/(m.K)</td>
</tr>
<tr>
<td>$L$</td>
<td>plate length, m</td>
</tr>
<tr>
<td>$L_c$</td>
<td>characteristic length, m</td>
</tr>
<tr>
<td>$L_{path}$</td>
<td>flow path length, m</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow rate, kg/s</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure, Pa</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$Q$</td>
<td>Cooling capacity, W</td>
</tr>
<tr>
<td>$q''$</td>
<td>heat flux, W/m$^2$</td>
</tr>
<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>$S$</td>
<td>tube spacing, m</td>
</tr>
<tr>
<td>$Sv$</td>
<td>Svelteness number</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature, K</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity vector, m/s</td>
</tr>
<tr>
<td>$V_{flow}$</td>
<td>flow volume, $m^3$</td>
</tr>
<tr>
<td>$W$</td>
<td>plate width, m</td>
</tr>
<tr>
<td>$\dot{W}$</td>
<td>pumping power, W</td>
</tr>
</tbody>
</table>

### Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta p$</td>
<td>pressure drop, Pa</td>
</tr>
<tr>
<td>$\delta$</td>
<td>thickness, m</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>emissivity</td>
</tr>
<tr>
<td>$\theta$</td>
<td>non-dimensional temperature</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density, $kg/m^3$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity, Pa.s</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity, $m^2/s$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan–Boltzmann’s constant, W.$m^{-2}.K^{-4}$</td>
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### Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>$amb$</td>
<td>ambient</td>
</tr>
<tr>
<td>$ave$</td>
<td>average</td>
</tr>
<tr>
<td>$in$</td>
<td>inlet</td>
</tr>
<tr>
<td>$max$</td>
<td>maximum</td>
</tr>
<tr>
<td>$n$</td>
<td>number of meshing element or tube</td>
</tr>
<tr>
<td>$nls$</td>
<td>analytical solution</td>
</tr>
<tr>
<td>$out$</td>
<td>outlet</td>
</tr>
<tr>
<td>$p$</td>
<td>plate</td>
</tr>
<tr>
<td>$pm$</td>
<td>plate mean</td>
</tr>
<tr>
<td>$r,c$</td>
<td>radiation, convection</td>
</tr>
<tr>
<td>$r$</td>
<td>ratio</td>
</tr>
<tr>
<td>$t$</td>
<td>tangential</td>
</tr>
<tr>
<td>$w$</td>
<td>water</td>
</tr>
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</table>

### Superscripts

<table>
<thead>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>~</td>
<td>dimensionless</td>
</tr>
<tr>
<td>–</td>
<td>average value</td>
</tr>
</tbody>
</table>
1. Introduction

Because their energy consumption is lower than air systems [1-4], radiant cooling panels are increasingly becoming solutions to improve indoor comfort. The panels are made of a hydraulic network attached to the upper side of a ceiling-suspended metallic plate. To provide adequate space cooling, water flows through this network to extract the heat received by the plate facing the surrounding ambient. Two modes of heat transfer occur simultaneously between the plate and its surrounding: radiation (the dominant) and convection.

Radiant cooling panels are designed to remove sensible heat only. As a consequence, ventilations and latent loads requirements are met by coupling with air handling units, which still have to be implemented. This raises the crucial drawback of cooling panels, namely, the risk of condensation. To avoid this issue, water should be supplied to the panel at a temperature sufficiently higher than the dew point temperature of the air in the conditioned space. However, increasing the water temperature also limits the cooling power, making the cooling panels more suitable for indoor spaces with low thermal loads [1, 3-5].

Much work has been devoted to radiant panels to evaluate their cooling performance and thermal comfort improvement at the room level. By this we mean, the focus has generally been about whether radiant panels have the potential of providing adequate thermal comfort or not, how energy efficient these systems are, compared to air systems, etc. At the panel scale, however, further studies are necessary on the flow network to extend the feasibility at larger scales as suggested by [6, 7]. In terms of the hydronic network, the serpentine flow layout has been widely employed [2, 8-16]. In this flow configuration, the working fluid circulates across the panel through a single long path composed of straight and U-turn segments connected in series. The
drawback of this flow structure is the high pressure drop across the flow network and the associated high pumping power. In other thermal systems, different flow patterns have shown the potential of overcoming this issue: for example, in a solar thermal collector, a harp (canopy-to-canopy) configuration, where the flow is guided through parallel lines, was found to improve the overall efficiency of the system [17]. In such design, the flow path is shortened, and therefore less pressure drop is encountered compared to the serpentine case for the same amount of fluid.

In 1996 Bejan introduced the Constructal law to predict the shape of a street network for smoothening the flow of people. He determined the path that connects any point of a locality to a common terminus to shorten the traveling time. The network was morphing under constraints [18]. In recent years there has been a growing interest in exploring original flow architectures to improve the overall performance of thermal systems. The Constructal law has been extending since its emergence into the field of thermodynamics. Fluid flow and heat transfer [19-27], cooling of electronics components [28-34], cooling of photovoltaic modules [35], thermal energy storage [36-38], iron and steel production processes [39], etc. are applications areas of Constructal design to mention a few in the domain of thermal design. Very recently, the Constructal law was applied to improve the design at large scale of radiant enclosures in the context of industrial furnace [40].

We have seen that the Constructal Law as a nature-inspired flow system design approach is expanding into broader engineering systems. It is essential to explore the competence of Constructal flow patterns in a wider range of engineering applications, at different scales, to gain a more comprehensive understanding of the performance of such configurations. The Constructal law is not only a way to facilitate the design procedure of flow system, but it also allows us to predict its overall performance staying away from the usual trial and error behavior. In this
approach, the flow architecture is given the freedom to change in time to facilitate the flow of the currents traversing its boundaries (here heat and fluid).

A compact design that provides efficient performance from a thermo-fluid perspective and takes into account space limitation is important in thermal systems applications. In our previous work [41], applying a Constructal approach, we showed that a cooling panel equipped with canopy-to-canopy (dendritic) flow channels has the potential of enhancing the overall performance compared to the serpentine design. Morphing the aspect ratio of a rectangular panel to more compactness is also a way toward efficient design. A nearly square shape provides the panel with a higher cooling capacity and less pumping power.

In the present study and still in the context of building applications, we continue to apply the Constructal law as an innovative approach to predict the overall performance of a cooling panel equipped with different flow architectures. An almost square panel shape is investigated. For all the flow architectures considered in this work, the fluid volume is fixed. An analytical approach is applied for sizing the branching flow passages in accordance with the Constructal law. We aim to provide a fundamental discussion on the role of the flow network toward better panel design.

The flow architectures are categorized into two groups depending on the location of the inlet and outlet of the working fluid and further sub-categorized based on the flow arrangements. From a thermal perspective, a high cooling capacity and uniform panel surface temperature favor the occupant thermal comfort conditions. From a thermo-fluid point of view, the flow system design needs to satisfy the thermal duty with the minimum possible pumping power drawback. The impact of the water inlet Reynolds number on the thermal and hydraulic performance of the panel is reported.
2. Mathematical and numerical model

Figure 1 portrays a sketch of a radiant cooling panel. The coolant flows through the channels secured to the upper side of a horizontal metal plate to extract the heat absorbed by the underside of the plate. The top and sides of the panel are insulated while its radiant surface exchanges heat with an ambient at a controlled temperature. In the present study, a numerical model was developed to evaluate the thermo-fluid performance of a cooling panel furnished with aluminum plate and copper tubes in different configurations: serpentine and branching patterns.

![Insert Fig. 1](image-url)

The 3D steady state mass conservation, Navier-Stokes and energy conservation equations were solved accompanied by the imposed boundary conditions presented in Table 1. A perfect contact between the flow channels and the plate was assumed. The flow was laminar and incompressible. The fluid Prandtl number, \( Pr_w = \mu_w c_{p,w}/k_w \), was considered constant (about 7.7). It was also assumed that the solid properties are constant. The governing equations describing the physical model are given by Eqs. (1) to (4).

\[
\nabla \mathbf{u} = 0 \tag{1}
\]

\[
\rho_w (\mathbf{u} \nabla) \mathbf{u} = \nabla[-p \mathbf{I} + \mu_w (\nabla \mathbf{u})] \tag{2}
\]

\[
\rho_w c_{p,w} \mathbf{u} \nabla T + \nabla(-k_w \nabla T) = 0 \tag{3}
\]

\[
\nabla(-k_p \nabla T) = 0 \tag{4}
\]

where \( \mathbf{u} \) is the water velocity field (m), \( p \) is the pressure (Pa) and \( T \) is the temperature (K). \( \rho_w \), \( \mu_w \), \( c_{p,w} \) and \( k_w \) designate the density (kg/m\(^3\)), dynamic viscosity (Pa.s), specific heat (J/kg.K) and thermal conductivity (W/m.K) of water, respectively. \( k_p \) denotes the plate thermal conductivity (W/m.K).
Table 1 Boundary conditions

<table>
<thead>
<tr>
<th>Domain</th>
<th>Boundary</th>
<th>Flow condition</th>
<th>Heat Transfer condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>Walls</td>
<td>( \mathbf{u} = 0 )</td>
<td>( -\mathbf{n} \cdot \mathbf{q} = -\nabla_{t}(\mathbf{T}) )</td>
</tr>
<tr>
<td></td>
<td>Inlet</td>
<td>( \text{Re}_D = 500, 1500 )</td>
<td>( T_{\text{w, in}} = 15^\circ \text{C} )</td>
</tr>
<tr>
<td></td>
<td>Outlet</td>
<td>( p = 0 )</td>
<td>( -\mathbf{n} \cdot \mathbf{q} = 0 )</td>
</tr>
<tr>
<td>Plate</td>
<td>Top &amp; sides</td>
<td></td>
<td>( -\mathbf{n} \cdot \mathbf{q} = 0 )</td>
</tr>
<tr>
<td></td>
<td>Bottom</td>
<td></td>
<td>( -\mathbf{n} \cdot \mathbf{q} = q_{r,c}^{''} ) (( T_{\text{amb}} = 24^\circ \text{C} ))</td>
</tr>
</tbody>
</table>

Here, \( \text{Re}_D \) is the water Reynolds number at the inlet, \( \mathbf{n} \) is the normal vector on the boundary, \( \mathbf{q} \) is the conductive heat flux vector (W/m\(^2\)), \( \nabla_{t} \) is tangential gradient, \( \delta \) is the tube wall thickness (m), \( k_{\text{tube}} \) is the tube thermal conductivity (W/m.K), \( T_{\text{w, in}} \) the water inlet temperature (\(^\circ \text{C}\)), and \( q_{r,c}^{''} \) is the total heat flux received by the plate (W/m\(^2\)).

The flow was given a long enough entrance length to ensure fully developed laminar inflow conditions. Water was pushed through the panel tube with an average velocity, \( U_{\text{ave}} \) (m/s), corresponding to a Reynolds number estimated as

\[
\text{Re}_D = \frac{U_{\text{ave}}D}{v_{\text{win}}} \tag{5}
\]

where \( D \) is the inlet tube diameter (m).

The copper tube was modeled as a thermally thin layer wrapped around the water domain. This layer is treated as a boundary condition imposed around the fluid as presented in Table 1. As copper has a high thermal conductivity, the thermal resistance across the thin tube wall was
assumed insignificant. Therefore, to reduce computational efforts, only the tangential heat flux was considered, which is equivalent to the outgoing heat flux to the water domain [41, 42].

The sum of the radiation and convection heat exchanges between the radiant surface of the plate and its surrounding represents the total heat flux received by the panel, \( q^r_{r,c} \).

\[
q^r_{r,c} = \varepsilon_p \sigma (T^4_{\text{amb}} - T^4_p) + h (T_{\text{amb}} - T_p)
\]  

(6)

Here, \( h \) denotes the heat transfer coefficient for turbulent natural convection air flow as defined by Eq. (7) [43]:

\[
h = \frac{k}{L_c} 0.15 \text{Ra}^{1/3}_{L_c}
\]  

(7)

where \( L_c \) is a characteristic length scale depending on the geometry of the plate, and \( \text{Ra}_{L_c} \) is the Rayleigh number.

According to ASHRAE, in radiant systems, radiation share should account for at least 50% of the total heat transfer. ASHRAE also uses the following correlations to estimate the radiation and convection heat fluxes to a suspended radiant cooling panel [8]:

\[
q^r = 5 \times 10^{-8} \left[ \text{(AUST} + 273)^4 - (T_p + 273)^4 \right]
\]  

(8)

\[
q^c = 2.31 (T_{\text{amb}} - T_p)^{0.31} (T_{\text{amb}} - T_p)
\]  

(9)

where AUST is the area-weighted average temperature of all the uncontrolled surfaces in the conditioned space and \( T_p \), denotes the effective radiant panel surface temperature.

Our results are compared with these correlations in the discussion section, 4.4.

Even though, the present study is intended to provide a fundamental discussion and purely numerical, typical real conditions found in literatures were selected as operating conditions and
input parameters. The main parameters namely the plate surface area, thickness, tube diameter and the thermal emissivity of the radiant surface of the plate are defined based on recommended values found in literature. References [8, 9, 16] are examples. The flow volume and the aspect ratio of the plate are results obtained from our previous study, [41]. We studied the influence of the aspect ratio of the plate (values ranged from 0.24 to 2.79) on the performance of the panel for serpentine tube layout. In accordance with the Constructal law, we fixed the flow volume and worked on dendritic flow architectures for similar aspect ratios. The almost square aspect ratio (1.05), therefore, was selected in the present study after seeing it has the potential of providing more compact panel design. Table 2 summarizes the main characteristics of a panel equipped with a serpentine flow channel.

Table 2 Main characteristics of a panel equipped with a serpentine flow channel

<table>
<thead>
<tr>
<th>$V_{\text{flow}}$ (m$^3$)</th>
<th>$A_R$</th>
<th>$A_p$ (m$^2$)</th>
<th>$\delta_p$ (mm)</th>
<th>$D$ (mm)</th>
<th>$\varepsilon_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.64 x 10^{-4}</td>
<td>1.05</td>
<td>1.35</td>
<td>3</td>
<td>10.4</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Here, $V_{\text{flow}}$ is the flow volume, $A_R = W/L$ is the plate aspect ratio, $A_p = W \times L$ is the plate area, $\delta_p$ is the plate thickness, $D$ is the tube diameter and $\varepsilon_p$ is the emissivity of the radiant surface.

The simulations were conducted with a finite element analysis software [42]. When a geometry exhibits a symmetry, only half of the geometry was considered. Care was applied to the mesh structure to ensure that temperature and pressure (the main variables) are mesh independent as shown in appendix A.

3. Designs
The design procedure described in this section is geared towards improving the cooling panels performance from a thermo-fluid perspective. In this paper, we investigate the influence of six different flow structural designs on the overall performance of radiant cooling panels of fixed: flow volume, $V_{\text{flow}}$, plate aspect ratio, $A_R$, and plate area, $A_p$. The flow architectures were categorized into two groups, with respect to the location of the inlet and outlet of the cooling water: design group A and design group B.

### 3.1 Design group A

In this group, water enters from one side, circulates through the panel and exits on the opposite side. This type of flow designs were further classified, based on the flow arrangement, into serpentine, canopy-to-canopy and tree-shaped. In the serpentine configuration (Fig. 2 (a)) single sized tubes, equally spaced, were connected in series via U-turns, resulting in one long channel. The corresponding flow volume, $V_{\text{flow}}$, can be expressed as:

$$V_{\text{flow}} = \frac{\pi}{4} D^2 L_{\text{path}}$$

(10)

where $D$ is the tube diameter (m) and $L_{\text{path}}$ is the flow path length (m) which is equivalent to the total length of the flow channel.

A Constructal approach was used to size the branching flow arrangements: the canopy-to-canopy and the tree-shaped configurations shown in Fig. 2 (b) and (c), respectively. Constructal design is about giving the flow architecture the freedom to evolve under predefined constraints. This methodology allows the use of multiple tube sizes instead of one, as in the case of the serpentine design. The Constructal law also defines the svelteness, $S_v$, of any flow system as the ratio between its external and internal length scales. It was demonstrated that in a flow system which
Sv > 10, the local pressure losses are insignificant compared to the friction losses, and therefore can be neglected when sizing the flow channels [44, 45]. Sv is given here as

\[
Sv = L_{\text{path}} \left( \frac{W/L}{V_{\text{flow}}} \right)^{1/3}
\]

(11)

The degree of freedom, here the channel size, was limited to two diameters in the canopy-to-canopy design, Fig. 2 (b). Two additional diameters were added in the tree-shaped configuration, Fig. 2 (c). Both designs preserve Sv = 26. As described in [41], applying a Constructal approach for laminar flow in a canopy-to-canopy arrangement, the diameter ratio, \( D_r \), for minimum overall pressure is determined as

\[
D_r = D_1/D_2 = \left[ \frac{245S}{2(L - S)} \right]^{1/6}
\]

(12)

For the tree-shaped structure, such as the one given in Fig. 2 (c), [45] showed that for symmetric branching patterns \( D_r = D_l/D_{l+1} = 1.26 \).

### 3.2 Design group B

In this group, the inlet and outlets of the fluid are located on the same side of the panel. The main flow is fed to the panel from the left in Fig. 2 (d) to (f) through the inlet pipe located at the \( W/2 \). The flow afterward divides into two symmetric streams, each of which covering half of the plate. The two streams continue bifurcating, forming a tree-like flow structure. The return lines eventually are forced to exit the panel separately at the left side creating a counterflow arrangement. This group was sub-categorized in accordance with the flow outline after the branching of the main flow into: counterflow serpentine, counterflow canopy-to-canopy and counterflow tree-shaped designs as shown in Fig. 2 (d) to (f), respectively.
All the flow systems considered in this group possess a $Sv > 10$. The channel sizing was based on the Constructal law. Again, the diameter ratio was calculated for minimum overall pressure losses in laminar flow based on a fixed fluid volume. Two tube diameters were used in the counterflow serpentine structure and four channel sizes are found in counterflow tree-shaped. The two geometries preserve symmetric bifurcating patterns. The flow divides equally between the two branches after any bifurcation. Therefore, $D_r = 1.26$ applies as described in [45]. In the counterflow canopy-to-canopy, three degrees of freedom were established with three different tube diameters. In this configuration, the main flow splits in half feeding two tree-shaped matched canopy-to-canopy arrangements. The two diameter ratios for minimum pressure losses, $D_{r1/2}$ and $D_{r2/3}$ are given by Eqs. (13) and (14).

$$D_{r1/2} = \frac{D_1}{D_2} = \left[ \frac{4(W + 22S)}{W + 7S} \right]^{1/6} \quad (13)$$

$$D_{r2/3} = \frac{D_2}{D_3} = \left[ \frac{36(W + 7S)}{W + 22S} \right]^{1/6} \quad (14)$$

As an example, a description of the Constructal procedure of sizing the flow passages for the counterflow canopy-to-canopy configuration is given in appendix B. Table 3 provides details of all the flow architectures designs. The overall flow volume remains the one provided in Table 1.

Table 3 Summary of the flow architectures designs

<table>
<thead>
<tr>
<th>Flow architectures</th>
<th>$^aL_{\text{path}}$</th>
<th>Sv</th>
<th>$D_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serpentine</td>
<td>1.00</td>
<td>84</td>
<td>-</td>
</tr>
<tr>
<td>Canopy-to-canopy</td>
<td>0.31</td>
<td>26</td>
<td>1.61</td>
</tr>
</tbody>
</table>
4. Results and discussion

In this work we aim to assess the combined performance of a cooling panel, equipped with different flow architectures, from a thermal and fluid standpoints. The performance was evaluated for \( \text{Re}_{\text{in}} = 500 \) and 1500 while the panel provided cooling to an ambient at \( T_{\text{amb}} = 24^\circ\text{C} \). The results are compared in terms of the temperature distribution on the radiant surface of the panel as well as the total heat absorbed by the panel. The water pressure drop across the panel, the ensuing pumping power and the overall efficiency of the panel are also assessed.

4.1 Temperature distribution

To describe the temperature of the plate, we propose to consider the dimensionless temperature \( \theta_p \), which relates the local temperature of plate surface, the water temperature and the ambient temperature.

\[
\theta_p = \frac{T_p - T_{\text{w,in}}}{T_{\text{amb}} - T_{\text{w,in}}} \quad (15)
\]
Of course, the temperature of the plate is inhomogeneous. As shown in Fig. 3, concentrations of cold region are clearly seen near the water inlet while some hot spots are observed near the edges of the plate. The highest $\theta_p$ value for all the cases is 0.55. It is obtained with the tree-like flow network for Re$_D$ = 500. Due to the heat extracted by the cooling water, the temperature of the plate rises in the direction of the flow, which is clearly visible for all the cases. The influence of the Reynolds number is visible: as the mass flow rate increases, the temperature of the panel is getting colder and the hot spots are strongly reduced.

In order to refine the analysis, we propose to rely on the temperature distribution of the plate by means of $A_r(\theta_p)$, as presented in Fig. 4. $A_r$ is a dimensionless value defined as the ratio of the surface area, $A_{\theta_p}$, which temperature is less than or equal to $\theta_p$, to the total area of the plate, $A_p$.

$$A_r = \frac{A_{\theta_p}}{A_p} \quad (16)$$

The temperature distribution follows a S-curve for all cases. A steeper curve means that the distribution is more uniform. A clear trend is noticeable when increasing Re$_D$ to 1500. All the curves shifted to the left compared to the results at Re$_D$ = 500, as the plate is colder for a higher mass flow rate. In terms of temperature distribution, at Re$_D$ = 500, the counter flow serpentine and counterflow canopy-to-canopy designs in group B provided better temperature uniformity compared to all the designs. While the counterflow serpentine resulted in the hottest plate surface among all the flow arrangements, the counterflow provided the coldest plate surface. At Re$_D$ = 1500, however, the effect of the flow architecture become less significant. Yet, the counterflow designs seem to provide a steeper (more uniform) temperature distribution and the tree-shaped design in group A leads to a more flattened distribution.
As the tubes do not cover the edges of the plate, the fluid have less access to the surface regions near the edges. As a result, these regions are relatively hotter than the rest of the surface, especially near the outlet of the flow. To get more insight on the temperature distribution, we propose to observe how the temperature is spread out over 90% of the radiant surface of the plate, \( S_{90\%} \). As shown in Fig. 5 (a), \( S_{90\%} \) represents the difference between the two temperature extremes (the maximum limit, \( \theta_{0.95} \), and the minimum limit, \( \theta_{0.05} \)). This index signifies that only 10% of the data from the total surface area of 1.35 (m²) is excluded. This portion may corresponds to area near the edges.

We note that at \( \text{Re}_D = 500 \), \( S_{90\%} \) for the serpentine and canopy-to-canopy designs is about 0.30 whereas it is wider for tree-shaped flow arrangement, 0.37. For the flow layouts in group B, \( S_{90\%} \) was in the range of 0.25 for the counterflow designs of the serpentine and tree-shaped. The counterflow canopy-to-canopy design leads to lower temperature spread, \( S_{90\%} = 0.19 \). At this \( \text{Re}_D \) value, the mean temperature of the plate surface, \( \theta_{pm} \), has a similar value for all the cases. We highlight that with increasing the Re range to 1500, as the plate surface becomes colder, the temperature spread decreases for all the designs. The discrepancy in \( S_{90\%} \) diminishes between the serpentine and canopy-to-canopy architectures in group A and counterflow designs of the serpentine and tree shaped in group B, \( S_{90\%} = 0.20 \). The widest \( S_{90\%} \) is observed for the tree-shaped flow arrangement, 0.31, while the narrowest temperature spread, \( S_{90\%} = 0.15 \), is obtained in the case of the counterflow canopy-to-canopy configuration. We found values for \( \theta_{pm} \) within the range of 0.18 for all the considered designs.
Overall, we conclude that the average temperature of the plate is little sensitive to the flow architecture and depends mostly on the Reynolds number. More variations were observed in terms of temperature uniformity. The best results (low and uniform temperature) were obtained with the counterflow canopy-to-canopy design.

4.2 Pressure drop features

Figure 6 shows, in dimensionless form, the pressure drop, $\tilde{\Delta p} = \Delta p/\Delta p_{\text{max}}$, for the different flow designs plotted as a function of the associated $\tilde{L}_{\text{path}} = L_{\text{path}}/L_{\text{path max}}$. To compare the performance of the different designs, the maximum pressure drop, $\Delta p_{\text{max}}$, and the longest flow path length, $L_{\text{path max}}$, are taken as references. Both $\Delta p_{\text{max}}$ and $L_{\text{path max}}$ are associated with the serpentine flow arrangement when $Re_D = 1500$. Please, note the use of a logarithmic scale for the y axis. At $Re_D = 500$, the serpentine flow layout in group A results in the highest $\tilde{\Delta p}$. The least pressure drop drawback is offered by the Constructal design of the canopy-to-canopy arrangement in the same group. Compared to the serpentine design, this is mainly due to the 70% decrease in $\tilde{L}_{\text{path}}$. The results also reveal that although the canopy-to-canopy and the tree-like structures possess identical flow path length, the former bid 50% lower $\tilde{\Delta p}$. For the same flow volume, this is attributed to the shape of the network. The flow arrangement over the surface of the plate controls the size (diameter) of the flow passages which are the consequence of determined diameter ratio(s), $D_r$, discussed in Section 3. $\tilde{L}_{\text{path}}$ is shortened by 40% for the counterflow serpentine relative to the standard serpentine structure. As a result, the pressure drop is lowered by about 11%. However, $\tilde{L}_{\text{path}}$ for this counterflow design is 20% longer than that of
the counterflow canopy-to-canopy and counterflow tree-shaped ones in group B. This yields 3% higher \( \Delta \bar{p} \) compared to the former and 2% greater than the latter.

As shown in Fig. 6, along with the increase of the Reynolds number, the pressure drop also increases to a great extent for all the designs. At \( \text{Re}_D = 1500 \), the data exhibits the same tendency compared with the results at \( \text{Re}_D = 500 \): the minimum pressure drop is offered by the canopy-to-canopy design and the maximum is associated with the serpentine flow layout. Also, the tree-shaped arrangement provides higher \( \Delta \bar{p} \) than the canopy-to-canopy one. The result from the counterflow structures of the serpentine, canopy-to-canopy and tree-shaped in group B are closed to that of the tree-shaped design in group A.

4.3 Overall performance

To further evaluate the performance of the panel, three other variables were computed:

- \( Q \), which denotes the cooling capacity of the panel (total absorbed sensible heat) and defined as \( \bar{q}_{r,c} \times A_p \). Here \( \bar{q}_{r,c} \) represents the average of the local fluxes obtained from Eq. (6).
- \( \dot{W} \), which is the required pumping power to drive the water across the panel expressed as \( \Delta \bar{p} \dot{m}/\rho \).
- \( \eta \), defined as the ratio between \( Q \) and \( \dot{W} \), so that it represents the efficiency of the system.

Figure 7 portrays \( \bar{Q}(\bar{W}) \). Here \( \bar{Q} \) denotes the ratio of a given \( Q \) to the maximum obtained value, \( Q_{\text{max}} \), and similarly \( \bar{W} \) is defined as \( \dot{W}/\dot{W}_{\text{max}} \). Here, \( Q_{\text{max}} \) is achieved with the counterflow canopy-to-canopy configuration whereas \( \dot{W}_{\text{max}} \) corresponds to the serpentine design. The two
values are obtained when $\text{Re}_D = 1500$. The graph indicates that at $\text{Re}_D = 500$, for the designs in group A, the canopy-to-canopy and the tree-shaped arrangements provide a slightly higher thermal power and expressively lower pumping power than the serpentine design. Therefore, the overall panel efficiency, $\eta$, is significantly higher. For the flow structures in group B, the data suggest that in terms of $\tilde{Q}$, the designs yield similar performance compared to the designs in group A. Yet with respect to $\tilde{W}$, the canopy-to-canopy and tree-shaped layouts in group A result in superior performance than all the designs in group B. Among all the designs, the most performant flow architecture is the canopy-to-canopy design in group A with $\eta = 0.69$. The standard serpentine flow layout is the least efficient with the lowest $\eta$, 0.1, as it results in the maximum $\tilde{W}$.

Expectedly, increasing the Reynolds number to 1500 enhances the heat transfer performance for all the designs compared to the results at $\text{Re}_D = 500$. Nevertheless, this comes at the expense of an additional pressure drop charge. For instance, $\tilde{Q}$ for the canopy-to-canopy is boosted by 17%. This enhancement is accompanied by a 27% augmentation in $\tilde{W}$. Therefore, $\eta$ declined by a factor of 11. The results are in line with the findings at $\text{Re}_D = 500$. $\tilde{Q}$ is comparable for all the designs. The counterflow canopy-to-canopy offers marginally the highest value. For the two Reynolds values considered, in all the designs, the radiation accounted for about 60% of the absorbed heat flux by the radiant surface of the plate. Measured against the serpentine flow design, the branching structures reduce the pumping requirements. The canopy-to-canopy layout is the most efficient and the serpentine is the poorest design. A global summary of the results is found in Table 4 (a) and (b).
Table 4 (a) Summary of the results at $Re_D = 500$

<table>
<thead>
<tr>
<th>Flow architectures</th>
<th>$\theta_{pm}$</th>
<th>$S_{90%}$</th>
<th>$\tilde{Q}$</th>
<th>$\tilde{W}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serpentine</td>
<td>0.33</td>
<td>0.30</td>
<td>0.78</td>
<td>0.06</td>
<td>0.10</td>
</tr>
<tr>
<td>Canopy-to-canopy</td>
<td>0.30</td>
<td>0.29</td>
<td>0.82</td>
<td>0.01</td>
<td>0.69</td>
</tr>
<tr>
<td>Tree-shaped</td>
<td>0.31</td>
<td>0.37</td>
<td>0.81</td>
<td>0.01</td>
<td>0.63</td>
</tr>
<tr>
<td><strong>Group B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>counterflow serpentine</td>
<td>0.34</td>
<td>0.24</td>
<td>0.77</td>
<td>0.03</td>
<td>0.21</td>
</tr>
<tr>
<td>counterflow canopy-to canopy</td>
<td>0.29</td>
<td>0.19</td>
<td>0.83</td>
<td>0.02</td>
<td>0.32</td>
</tr>
<tr>
<td>counterflow tree-shaped</td>
<td>0.30</td>
<td>0.25</td>
<td>0.82</td>
<td>0.03</td>
<td>0.27</td>
</tr>
</tbody>
</table>

\[
\theta_{pm} = \frac{T_{pm} - T_{w,in}}{T_{amb} - T_{w,in}} \quad \tilde{Q} = \frac{Q}{Q_{max}} \quad \tilde{W} = \frac{\dot{W}}{\dot{W}_{max}}
\]

Table 4 (b) Summary of the results at $Re_D = 1500$

<table>
<thead>
<tr>
<th>Flow architectures</th>
<th>$\theta_{pm}$</th>
<th>$S_{90%}$</th>
<th>$\tilde{Q}$</th>
<th>$\tilde{W}$</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group A</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Serpentine</td>
<td>0.19</td>
<td>0.20</td>
<td>0.96</td>
<td>1.00</td>
<td>0.01</td>
</tr>
<tr>
<td>Canopy-to-canopy</td>
<td>0.18</td>
<td>0.19</td>
<td>0.97</td>
<td>0.13</td>
<td>0.06</td>
</tr>
<tr>
<td>Tree-shaped</td>
<td>0.19</td>
<td>0.31</td>
<td>0.96</td>
<td>0.18</td>
<td>0.04</td>
</tr>
<tr>
<td><strong>Group B</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>counterflow serpentine</td>
<td>0.19</td>
<td>0.19</td>
<td>0.96</td>
<td>0.38</td>
<td>0.02</td>
</tr>
<tr>
<td>counterflow canopy-to canopy</td>
<td>0.16</td>
<td>0.15</td>
<td>1.00</td>
<td>0.29</td>
<td>0.03</td>
</tr>
<tr>
<td>counterflow tree-shaped</td>
<td>0.17</td>
<td>0.21</td>
<td>0.99</td>
<td>0.37</td>
<td>0.02</td>
</tr>
</tbody>
</table>
4.4 Discussion

The present work intended to provide a fundamental discussion on a single panel design for cooling, in the light of the Constructal law approach. Typical real conditions found in literatures were selected as operating conditions and input parameters. While the panel operates in an indoor space in which the air and all the surfaces are at a fixed temperature, $T_{\text{amb}} = 24^\circ\text{C}$, the water enters the panel at $15^\circ\text{C}$ to control its radiant surface temperature. This temperature difference drives the heat exchange between the radiant surface of the panel and the conditioned surrounding. Depending on the flow rate (Reynolds number), the mean radiant surface temperature varies between $16.4^\circ\text{C}$ and $18.1^\circ\text{C}$, which is more suitable for a rather dry indoor environment.

As shown in Eq. (6), the total heat flux received by the plate is a combination of radiation between the panel surface and the surrounding surfaces in addition to heat convection with air. Under the considered operating conditions, presented in Table 1, radiation accounted for about 60% of the absorbed heat flux by the radiant surface of the plate. Our results are in agreement with ASHRAE’s definition of radiant systems and correlations, Eq. (8) and Eq. (9). This also corresponds to typical values found in previous work. For example, Miriel et al. [1] showed that radiation accounted for 2/3 of the total heat exchange.

The temperature spread over the radiant surface of the panel replicates a S-curve shape, Fig. 4. The Constructal law explained this S-shaped phenomenon that naturally occurs [47]. When the cooling fluid enters (invades) the panel, a slow growth in the temperature spread over the radiant surface is observed. As the flow moves forward across the panel, the growth happens faster before it slows down again as the working fluid abandons the panel territory. The temperature
spreads over the plate surface along the flow direction and in the direction transversal. As the operating conditions namely the water inlet and the ambient temperatures are kept constant, the scales of the S-cure here for a given panel configuration depends on Reynolds number, the flow rate in another word. As portrayed in Fig. 3 and Fig. 4, the temperature spread happened faster at $Re_D = 1500$ compared to the case at $Re_D = 500$.

The flow uniformity is an important factor that would affect the performance of any thermal system; radiant panels are not an exception. Applying a Constructal approach, the size of the flow channels in tree-like architectures was analytically determined. Thus, the flow distribution was observed in the smallest branches in all the designs. Relying on the method of standard deviation, as presented in appendix C, the numerical experiments validates the theoretical predictions in terms of flow distribution.

Radiant panel should be design to provide adequate indoor cooling. Yet, this is only one aspect of a broader picture. Our results, for a single panel unit, raise the question of how the panel should be connected and distributed over the ceiling. Besides, how many units are really needed? In practice, for a single conditioned space within a large building envelop, usually multiple panels are connected in series and/or in parallel covering the entire or maybe a large portion of the ceiling surface. Water at high flow rate, therefore, is pushed through the panel to cover the large radiant surface so that the temperature spread is uniform. This comes at the expense of an additional pumping power drawback. Therefore, documenting the present findings serve as guidance in the search for better panel design, promoting additional energy savings while maintaining adequate indoor comfort.
Conclusion

This paper investigated the global performance of a suspended radiant cooling panel. Typical and innovative flow arrangements were deployed to look into the efficiency of a radiant panel from thermal and hydraulic perspectives. We described the Constructal approach as a way to improve existing design practices of radiant panels. Under the assumed operating conditions, our work led us to conclude that:

- The average temperature of the plate is little sensitive to the flow architecture and depends mostly on the Reynolds number.
- The counterflow architectures lean toward better temperature uniformity. The best results (low and uniform temperature) are obtained with the counterflow canopy-to-canopy design.
- All the branching flow designs offer significantly less pressure drops compared to the serpentine flow arrangement. The canopy-to-canopy (dendritic) provides the least pressure drop penalty.
- For a given Reynolds number, the cooling capacity is comparable for all the designs.
- Increasing the Reynold number enhances the cooling capacity. Yet, this comes at the expense of an additional pumping power demand. The overall efficiency of all the Constructal designs is superior to that of the standard serpentine one. The most performant is the dendritic canopy-to-canopy design.

Acknowledgment

The authors would like to acknowledge the Libyan scholarship program for M. Mosa’s support.
Appendix A, Mesh independence study:

Care was applied to the mesh structure to ensure that the main variables are mesh independent throughout our study. For illustration, the case of the counterflow canopy-to-canopy portrayed in Fig. 2(e) is presented for $Re_D = 1500$. Only half of the considered geometry was modeled as it possess a plane of symmetry passing at $W/2$.

To generate the mesh displayed in Fig. A1, first, a free tetrahedral element was applied to generate unstructured mesh at the junctions. Second, a free triangular element was used to mesh the interconnecting faces. Afterward, swept function was utilized to create structured mesh through the straight flow passages. Finally, a free triangular element was chosen again to mesh the upper side of the plate, which eventually was swept through the plate domain. The size of meshing elements was varied to control the total number of elements, $N$, for the entire geometry between $6.33 \times 10^4$ and $3.93 \times 10^5$. The calculation time varies accordingly between 3 to 25 minutes.

The impact of the mesh on the temperature distribution on the radiant surface of the plate, $A_r(\theta_p)$, in addition to the pressure drop, $\Delta p_N$, and the increase in the water temperature, $\theta_{out}$ was evaluated. As presented in Fig. A2, in terms of temperature at the plate and waterside is not influenced by $N$. The pressure drop, however, seems to be very little sensitive to the mesh. The relative difference in $\Delta p_N$ between any two successive meshes, however, did not exceed 3%. A
converged solution, therefore, could be achieved with small number of meshing elements without scarifying the accuracy of the results. Therefore, here we chose the computationally least expensive option.

**Appendix B,** Constructal channel sizing for the counterflow canopy-to-canopy design shown in Fig. 2 (e):

Assuming a uniform flow distribution, the main flow bifurcates into two branches with identical flow rate, \( \dot{m}/2 \). As the flow moves forward, each division evenly feeds 6 equally spaced parallel lines, \( \dot{m}/12 \) for each segment. For this configuration, \( S_v = 35 (>10) \) and thus the friction losses dominates the overall pressure losses [44, 45]. \( \Delta p \) depends on the flow rate, channels sizes and lengths. For laminar flow in round tubes, the ratio \( \Delta p/\dot{m} \) can be expressed as [46]

\[
\frac{\Delta p}{\dot{m}} \sim \frac{(L - S)}{D_1^4} + \frac{(1 + \sum_{i=1}^{5} i/12)S}{D_2^4} + \frac{(W - 4S)}{24D_3^4}
\]

(A1)

For this flow architecture, the volume allocated to the flow, \( V_{\text{flow}} \) is:

\[
V_{\text{flow}} \sim (L - S)D_1^2 + (W + 22S)D_2^2 + 6(W - 4S)D_3^2
\]

(A2)

Lagrange multipliers is invoked to find the extremum of \( \Delta p/\dot{m} \) where \( V_{\text{flow}} \) is fixed. This corresponds to finding the extremum of the aggregate function \( \varphi = \Delta p/\dot{m} + \lambda(V_{\text{flow}}) \), where \( \lambda \) is the Lagrange multiplier [45]. The optimum channels sizes for minimum flow resistance, therefore, are obtained as

\[
D_{1/2} = \frac{D_1}{D_2} = \left[ \frac{4(W + 22S)}{W + 7S} \right]^{1/6}
\]

(A3)

\[
D_{2/3} = \frac{D_2}{D_3} = \left[ \frac{36(W + 7S)}{W + 22S} \right]^{1/6}
\]

(A4)
Appendix C, Flow uniformity

The diameters ratios for minimum flow resistance were determined analytically following a Constructal approach assuming a perfect flow rate distribution. To validate this assumption, the flow rate distribution in the small parallel branches in the numerical solution was assessed using the method of the standard deviation. The standard deviation of the obtained value from the analytical one, \( \varphi \), is defined as

\[
\varphi = \left[ \frac{\sum_{i=1}^{N} (\tilde{m}_i - \tilde{m}_{\text{nls}})^2}{N - 1} \right]^{1/2}
\]

where \( N \) is the number of the small parallel branched tubes, equal 4 and 6 for the tree-shape arrangement and the counterflow canopy-to-canopy design, respectively. \( \tilde{m}_i = \dot{m}_i / \dot{m}_{\text{in}} \) represents the mass flow rate in the \( i \)th branch in dimensionless form, \( \dot{m}_{\text{in}} \), and \( \tilde{m}_{\text{nls}} = \dot{m}_{\text{nls}} / \dot{m}_{\text{in}} \) is the mass flow rate, also dimensionless, corresponding to the analytical solution. \( \dot{m}_i \) and \( \dot{m}_{\text{nls}} \) are in turn the mass flow rate (kg/s) in the \( i \)th tube and the analytical flow rate value, respectively.

\( \varphi = 0 \), means perfect agreement between the numerical and analytical solutions. Higher \( \varphi \) values, indicates a degree of divergence. The flow distribution in the designs with symmetrical bifurcations, \( D_r = 1.26 \) applies all the way along the flow direction, precisely replicate the analytical values, \( \varphi \approx 0 \). A maximum deviation from the analytical solution of 4% was noticed in the canopy-to-canopy design, Fig. 2 (b), when \( \text{Re}_D = 1500 \). In this design, higher flow rate is observed in the farthest branches along the entrance tube. The trend is similar in the small canopy-to-canopy in the counterflow canopy-to-canopy design, Fig. 2 (e). However, the mass
flow rate is deviated from the analytical value only by about 2% for both Reynolds number values considered in this work. For illustration purposes, only the results for the tree-shaped, Fig. 2 (c), and counterflow canopy-to-canopy, Fig. 2 (e), are presented in Fig. C1.

REFERENCES


**Figure captions**

Figure 1  Sketch of a radiant cooling panel.

Figure 2  Flow architectures: (a) serpentine, (b) canopy-to-canopy, (c) tree-shaped, (d) counterflow serpentine, (e) counterflow canopy-to-canopy and (f) counterflow tree-shaped.

Figure 3  Temperature map of the radiant surface of the cooling panel.

Figure 4  Temperature distribution on the radiant surface of the cooling panel: (a) design group A and (b) design group B.

Figure 5  Temperature spread over 90% of the radiant surface: (a) definition and (b) variation with the main temperature of the radiant surface of the panel.

Figure 6  Pressure drop variation with the flow path.

Figure 7  Global performance map.

Figure A1  Example of mesh.

Figure A2  Effect of mesh on the main results for the counterflow design at Re_D = 1500: (a) variation of the temperature distribution on the radiant surface of the panel with the number of meshing element (b) impact of the mesh on the pressure drop and the increase of the water temperature.

Figure C1  Flow distribution evaluation: (a) tree-shaped arrangement (b) counterflow canopy-to-canopy design.