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1 Expansion modelling based on cracking induced by the 2 formation of new phases in concrete

3
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10 Abstract

11 Unexpected expansion can be a cause of damage in quasi-brittle materials such as
12 concrete, fired clays or rocks. Such expansions can be due to chemical reactions (alkali-
13 silica reaction – ASR, sulfate attack) or physical phenomena (frost, transport of fluids).
14 Expansion is caused by two main mechanisms: matrix deformation due to the pressure
15 initiated by the formation of new phases in the porosity of the material, and matrix
16 cracking, which increases the apparent swelling of the matrix. In this work, a
17 poromechanical model is proposed to reproduce expansion in materials resulting from
18 single or multiple pressures. In this model, plastic deformation is used to represent
19 permanent strain due to diffuse cracking induced by the pressure. In the case of
20 expansion in quasi-brittle materials, the permanent deformations induced by the pressure
21 are anisotropic to take account of the impact of the initial material anisotropy and stress
22 anisotropy on cracking. The model is then coupled with a damage model and used to
23 evaluate the opening of localized cracks due to a pressure gradient in a laboratory
24 specimen.

26 Keywords

27 Anisotropy, cracking, expansion, plasticity, pressure.

28
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30 **1. Introduction**

31 Damage due to expansion in materials decreases the service life of structures.
32 Unexpected expansion can be responsible for cracking in concrete, fired clays or other
33 quasi-brittle materials and mechanical modelling should be able to evaluate their
34 behaviour at material and structure scales. External or internal chemical attacks (Alkali-
35 Silica Reaction – ASR [1], Delayed Ettringite Formation – DEF [2], external sulfate
36 attack [3], expansion of resins embedded in quasi-brittle materials [4]) or physical
37 phenomena (frost [5]) can be the cause of such swelling and lead to a decrease in the
38 strength of the materials affected. For quasi-brittle materials, irreversible deformation due
39 to cracking induced by pressure is the main cause of the positive strain of expansion. The
40 balance between local stresses around expansive new phases and stresses due to external
41 loading leads to anisotropic cracking. Given the dependence of the pressure on
42 environmental conditions, the development of such expansions can also lead to
43 deformation gradients both in real structures and in specimens studied under laboratory
44 conditions. The consequence is the opening of localized cracks on the skin of the
45 concrete.

46 Modelling should be able to distinguish and to assess the contribution of each
47 mechanism: pressure, diffuse cracking due to material damage, and localized cracking
48 due to pressure gradient.

49 In this work, a poromechanical model is proposed and used to quantify expansion in
50 concrete. First, the determination of the pressure according to the volume increase of new
51 phases in the material is described within the framework of poromechanics [6], [7]. In
52 particular, the case of alkali-silica reaction, for which pressure occurs in inclusions
53 (aggregate) embedded in the material (concrete) is addressed as a case for which up-
54 scaling and multiple pressures should be considered. Plasticity is commonly used to
55 represent permanent strain of quasi-brittle materials [8], [9]. In the present work, plastic
56 deformation is adopted to model the permanent strain due to the diffuse cracking caused
57 by the pressure. The criteria associated with these deformations are orthotropic so as to

58 consider the impact of an anisotropic stress state on expansions [10], [11]. The model is
59 then used to reproduce the swelling anisotropy induced either by the anisotropic initial
60 strength of certain materials, or by the anisotropy due to the stress state. The analysis
61 highlights the importance of cracking in the mechanisms of expansion in concrete.

62 **2. Pressure**

63 Because of their low strength in tension, the elastic deformation of quasi-brittle materials
64 under internal pressure is small. Cracking thus plays an important role in deformations
65 and has to be accurately determined. After cracking, the evolution of irreversible
66 deformation under the effect of expansive phases is still driven by their pressures.
67 Therefore, the same care has to be taken when evaluating pressure or anisotropic
68 permanent strains corresponding to the induced cracking.

69 ***2.1 Pressure due to the formation of new phases***

70 The pressure induced by the formation of new phases in materials depends on the amount
71 and the nature of the reactive components [12]–[21], on the structure of the material
72 porosity [22]–[25] and on the capability of the porous material to tolerate induced
73 tension. The interactions between the swelling inclusions and the matrix determine the
74 pressure developed in the swelling phase. In this work, the pressure, p_i , in the porous
75 material due to inclusion i is determined in the framework of poromechanics [6], [7]
76 through the following equation:

$$p_i = M_i \cdot (\varphi_i - \varphi_i^0 - \varphi_p - \varphi_{cr}) \quad (1)$$

77 with:

78 M_i the Biot Modulus related to the inclusions,

79 φ_i , the volumetric fraction of porosity filled by the inclusions related to the skeleton
80 deformation [23]. It can be assessed from the growth of the new phases in relation to
81 crystallization pressure [12], [13], [23] or to the volume produced by chemical reaction
82 [14], [26].

83 φ_i^0 is the volumetric fraction of new phases leading to little expansion during the reaction
84 for mechanical reasons (migration in porosity [25], [27]–[29], formation in pores too

85 large to induce pressure [14], [22], [30]) or chemical reasons (composition of the phases
86 [31]–[33] or non-expansive surface absorption [34]). It is taken to be proportional to the
87 pressure:

$$\varphi_i^0 = \varphi_{ref} \cdot \frac{p_i}{p_{ref}} \quad (2)$$

88 φ_{ref} is a reference volume. It corresponds to the volume of new phases formed for the
89 pressure p_{ref} that leads to the first cracking at material scale in free swelling conditions.
90 It can be related to the pore distribution [14], [27] or be calibrated to obtain the delay at
91 the beginning of the expansion curve.

92 φ_p is the volumetric deformation – due to elasticity – of porosity filled by the inclusion
93 of new phases, and delayed deformation (diffuse cracking due to the pressure and
94 structural cracks are not considered in this term but in φ_{cr} as explained below), which
95 can be calculated with b_i , the Biot coefficient related to the expansive inclusions:

$$\varphi_p = b_i \cdot (\varepsilon - \varepsilon_{cr} - \varepsilon_{an}) \quad (3)$$

96 Here, ε is the mechanical deformation of the porous material, and ε_{cr} its deformation due
97 to cracking induced by the pressure. The impact of ε_{cr} on pressure is considered in (1)
98 with an interaction coefficient of 1, and thus greater than the Biot coefficient used in (3).
99 ε_{an} is the deformation due to structural cracks that are not connected to the sites of
100 formation of inclusions and so cannot have a direct impact on the pressure.

101 In equation (1), φ_{cr} is the volume of diffuse cracks caused by the pressure; it is equal to
102 the volumetric deformation of the porous material due to the diffuse cracking ε_{cr} . In this
103 equation, for the sake of simplicity, this volume is assumed to be totally filled by the new
104 phases. The main assumption of the model is then that the new phases can migrate only
105 toward the cracks they have themselves created.

106 The role of φ_{cr} in Equation (1) is particularly important for the modelling of such
107 phenomena. If at least one direction is free of compressive stress in the material, the
108 pressure p_i cannot increase very greatly as cracking is possible perpendicularly to the free
109 direction. This is taken into account through φ_{cr} . If φ_{cr} increases, the stabilization of the
110 pressure p_i can be obtained. This stabilization of the pressure is necessary to obtain

111 realistic stress in cases of restraint, as observed in experimentally [30], [35]–[38]. This
112 point is highlighted in the part that focuses on cracking.

113

114 M_i , the Biot Modulus related to the inclusions, depends on the volume of new phases φ_i
115 and on the mechanical properties of the matrix (bulk modulus K_m) and the inclusions [6],
116 [7], [24]. m is the subscript related to the matrix, including solid and porosity that is not
117 filled by the new phase inclusions. It is evaluated with the following equation:

$$\frac{1}{M_i} = \frac{b_i - \varphi_i}{K_m} + \frac{\varphi_i}{K_i} \quad (4)$$

118 If the new phases are taken to be spherical and embedded in a homogeneously solid
119 matrix [24]; b_i , the Biot coefficient related to the inclusion, can be estimated using a
120 Mori-Tanaka scheme for the estimation of the bulk modulus, K , of the porous material
121 [39]:

$$b_i = 1 - \frac{K}{K_m} = \frac{\varphi_i(3K_m + 4\mu_m)}{(3K_m\varphi_i + 4\mu_m)} \quad (5)$$

122 with μ_m , the shear modulus of the matrix.

123 For simplicity, the Biot coefficient is evaluated in a linear context here, from the
124 mechanical properties and volumes of inclusions (Equation (5)) without consideration of
125 the shape of the porosity filled by the new phases. More precise quantification could be
126 obtained by taking account of the shape of filled cracks as proposed in [26], [27], [40]. In
127 the present work, the non-linearity due to the combination with diffuse cracking caused
128 by pressure, creep [41] and structural damage [42] is considered differently, by resorting
129 to damage theory [53].

130 The advancement of the new phases production can be evaluated with physicochemical
131 considerations or empirical laws. The aim of the paper is to focus on mechanical
132 considerations. As the chemical advancement is usually not impacted by mechanical
133 conditions in the case of the expansion due to new phases formation in concrete [43], the
134 chemical advancement can be separated of the mechanical problem. The equations
135 presented in this paper can thus be used whatever the approach applied to determine the
136 chemical advancement (theoretical or empirical approaches [12], [15], [17], [18], [20],
137 [21], [25], [44]).

138 The volumetric fraction of porosity filled by the inclusions can be considered as a data
 139 and its determination is summed up by the following equation:

$$\varphi_i(t) = n_i^{max} \cdot \varphi_i^{mol} \cdot A(t) \quad (6)$$

140 With n_i^{max} , the maximal number of moles of the new products which can be formed in
 141 the material, φ_i^{mol} , the molar volume of the new phases and $A(t)$, the advancement of the
 142 reaction.

143 **2.2 Case of porous materials embedded in another material**

144 In some cases, the expansive inclusions can be formed in porous materials embedded in
 145 other materials. For example, Alkali-Silica Reaction (ASR) expansion can occur in
 146 aggregates embedded in concrete [45]–[48]. In this situation, scale-up is necessary to
 147 evaluate, firstly, the impact of the formation of new phases on the lower scale matrix (LS
 148 in Figure 1, for example the aggregate) and, secondly, the impact of the pressure of the
 149 lower scale matrix on the macro scale (MS in Figure 1, for example, the concrete). The
 150 volumetric fraction of the embedded material per volume of the macro scale material is
 151 referenced as φ_a . Moreover, the material embedded in the macro scale material can have
 152 different sizes and volumetric fractions and lead to multiple pressures [4], [27]. The
 153 pressure of each fraction p_a is different and should be evaluated separately.

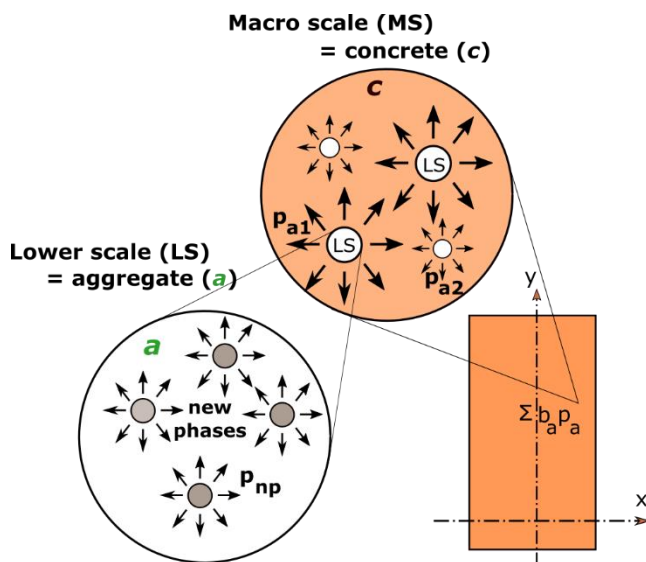


Figure 1: Porous material (Lower Scale, LS) embedded in another porous material (Macro Scale, MS)

154 The three equations of the multi-scale approach used to evaluate the pressure due to the
 155 new phases p_{np} on the lower scale matrix (Equation (7)), the resulting pressure of the
 156 lower scale p_a on the macro scale (Equation (8)) and finally the action of the multiple
 157 pressures p_a of the lower scale matrix on the macro scale (Equation (9)) are:

$$\begin{cases} p_{np} = M_{np} \cdot \left(\varphi_{np} - \varphi_{np}^0 - \varphi_p - \frac{\varphi_{cr}}{\varphi_a} \right) & \downarrow \text{From new phases to lower scale} & (7) \\ p_a = b_{np} \cdot p_{np} & \downarrow \text{From lower scale to macro scale} & (8) \\ \tilde{\sigma}_s = K_m \cdot (\varepsilon - \varepsilon_{an}) - \sum_a b_a \cdot p_a & & (9) \end{cases}$$

158 The interactions between the three equations and the different terms of the equations are
 159 explained in the three following subparts:

160

161 *Pressure of new phases (NP) on the lower scale (LS) material*

162 At the lower scale, the inclusion i ($i = np$ at this scale) is the new phases, and the matrix,
 163 m ($m = a$ at this scale) is the lower scale (LS) matrix. As the expansive inclusions are
 164 formed at the lower scale (Figure 1), the pressure induced by the formation of new phases
 165 on the LS matrix can be defined from Equation (7).

166 φ_{np} , φ_{np}^0 and φ_p are defined above and have to be determined per unit of volume of the
 167 lower scale matrix (here the aggregate). M_{np} and b_{np} related to the new phases are
 168 evaluated from Equations (4) and (5).

169 As the cracks induced by the pressure of the new phases are usually connected with the
 170 cracks at macro scale [48], the volume of cracks φ_{cr} in which the new phases can migrate
 171 is taken to be the crack volume available in the macro scale material. At the scale of the
 172 aggregate, this volume has to be normalized according to the volumetric fraction of
 173 aggregate φ_a in order to be obtained per unit of volume. Consequently, the impact of the
 174 crack volume on pressure is fully considered at this lower scale.

175

176 *Pressure of the lower scale (LS) material on the macro scale (MS) material*

177 At the macro scale (Figure 1), the inclusion i ($i = a$ at this scale) is the aggregate, and the
 178 matrix m ($m = c$ at this scale) is the concrete. M_a and b_a , related to the aggregate, are
 179 evaluated from Equations (4) and (5).

180 The material at macro scale subjected to the pressure of the aggregate is under tension.
 181 Because of its low tensile strength, the impact of its rigidity on aggregate pressure is
 182 negligible compared to the impact of the crack volume, which was considered at the
 183 lower scale. The pressure induced by the lower scale (aggregate) on the macro scale
 184 (concrete) can thus be evaluated from Equation (8).

185

186 *Consequence on pressure at the macro scale (MS) in case of expansive inclusions having*
 187 *different sizes*

188 As proposed in [7], [24], [26], the impact of multiple pressures on the macro scale
 189 material can be obtained by summing the pressure, taking Biot coefficients into account.

190 The constitutive equation for the mean stress $\tilde{\sigma}_s$ at macroscopic scale is then Equation
 191 (9).

192 In this case of multiple pressures, the volume of cracks of the macro scale material (φ_{cr}
 193 in equation (7)) has to be distributed among the different fractions of aggregates (φ_a). In
 194 this work, the distribution is assumed to be proportional to the volume of the new phases
 195 φ_{np_a} leading to cracking (so the volume of phases higher than φ_{np}^0 in equation (10)). For
 196 material containing several sizes, a , of aggregate, it is necessary to distinguish the new
 197 phases created by each aggregate: φ_{np_a} is the volumetric fraction of new phases created
 198 in aggregate a and $\varphi_{np_a}^0$ is the volumetric fraction of new phases necessary to start
 199 cracking in aggregate a . Thus, the volume of the crack induced by aggregate a is
 200 evaluated by (10):

$$\varphi_{cr}^a = \frac{\langle \varphi_{np_a} - \varphi_{np_a}^0 \rangle^+}{\sum_a \langle \varphi_{np_a} - \varphi_{np_a}^0 \rangle^+} \varphi_{cr} \quad (10)$$

201

202 For expansion in quasi-brittle materials, the model considers damage through a coupling
 203 with the modelling presented in [42]: the coupling method consists of affecting $\tilde{\sigma}_s$ by the
 204 damage according to the principle already presented in [11] (Equation (11)):

$$\sigma_s = \tilde{\sigma}_s (1 - D) \quad (11)$$

205 The damage variable, D , depends on the permanent strain related to the diffuse cracking
 206 and on the effective stress $\tilde{\sigma}_s$ due to the external loading.

207 Creep of the matrix, simultaneous with expansion, is also considered so as to obtain a
 208 realistic evaluation of concrete damage at micro and macro scales [11], [49]: ε_{an} is the
 209 inelastic strain due to creep and cracking at the macro-scale. Creep is considered through
 210 the model presented in [41] combined with the damage model proposed in [42] according
 211 to the combination method described in [11].

212 **3. Cracking**

213 **3.1 Diffuse cracking due to pressure**

214 In stress free conditions, the diffuse cracking induced by the pressure can be isotropic
 215 (sulfate attack [38]) or anisotropic (alkali-silica reaction [50], [51], and frost in fired clays
 216 [52]). In the absence of external loading, anisotropic expansion can be explained by the
 217 initial anisotropy of the materials, for instance by the shape and the orientation of the
 218 expansive inclusion (Figure 2). At the scale of the porous material, the mean stress field
 219 is homogeneous. For uniformly distributed spherical inclusions, no direction is
 220 preferential in stress free conditions (Figure 2). In this case, the cracking direction should
 221 be isotropic at the representative volume scale. For flattened inclusions oriented in a
 222 preferential direction, the stress induced by the pressure at the points with the smallest
 223 radius is larger than anywhere else (Figure 2). This leads to anisotropic cracking. The
 224 presence of water pockets under the expansive inclusions can also cause orientation of
 225 cracking [53]. In the absence of loading, this effect can be evaluated through the
 226 following anisotropic cracking criteria f_i according to f_t , the tensile strength of the
 227 porous material:

$$\begin{cases} f_I = k_I \cdot p_{np} - f_t \\ f_{II} = k_{II} \cdot p_{np} - f_t \\ f_{III} = k_{III} \cdot p_{np} - f_t \end{cases} \quad (12)$$

228 k_i are the stress concentration factors in the three main directions of the aggregate shape
 229 and p_{np} the pressure of the new phases produced in the embedded materials and
 230 responsible for the diffuse cracking.

231 If one of these criteria f_i becomes higher than zero, cracks appear along this direction. In
 232 the absence of loading, the cracks appear first in the plane perpendicular to the direction
 233 having the largest stress concentration factor.

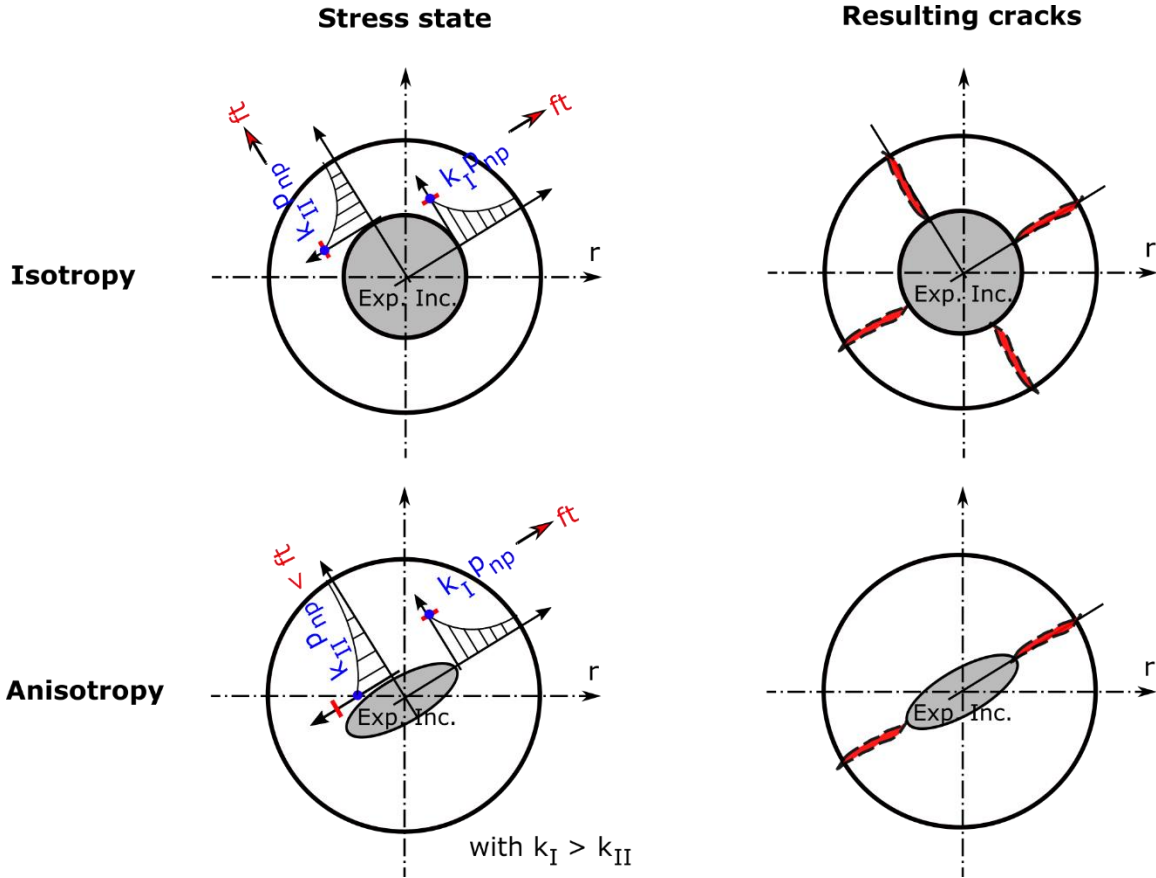


Figure 2: Ortho-radial stress state around spherical and penny-shaped inclusions from continuous mechanics and resulting cracks

234 With the simplified assumption of spherical inclusions, k_I is independent of the
 235 orientation and can be evaluated in a linear context from the proportion of new phases:

$$k_I = k_{II} = k_{III} = \frac{1 + 2 \cdot \varphi_{np_a}}{2 \cdot (1 - \varphi_{np_a})} \quad (13)$$

236 If the material is damaged by multiple pressures, the single stress used in the plastic
 237 criterion at the macro-scale has to consider all the crack initiation possibilities at smaller
 238 scales. For this purpose, the local stress is evaluated in proportion to the pressures

239 weighted by the volume of new phases φ_{np_a} leading to cracking (so the volume of
240 phases higher than φ_{np}^0).

241 According to the geometry of the inclusion, the stress concentration factor can be
242 dependent on the direction (Figure 2). For orthotropic inclusions, the ratio between the
243 stress concentration factors in the three main directions (I, II, III) of a penny shaped
244 inclusion a (Figure 2) is defined by equation (14):

$$k_{II} = k_{III} = \rho_{ani} \cdot k_I \quad (14)$$

245 ρ_{ani} is the rate of anisotropy (less than 1).

246 If ρ_{ani} is equal to 1, the cracking in stress-free conditions remains isotropic in the
247 absence of external loading. If ρ_{ani} is less than 1, the cracking in stress-free conditions
248 becomes orthotropic and occurs first in the plane perpendicular to the direction I
249 (Equation (14), Figure 2). This leads to anisotropic expansions.

250 **3.2 Anisotropy of cracking due to macroscopic stress state**

251 Under anisotropic mechanical loading, expansions are anisotropic [36], [37], [43], [54]–
252 [57]. For different natures of new phases (alkali silica gels or ettringite), a difference of
253 1 MPa in stress compared to the mean stress in the material is sufficient to cause
254 anisotropic expansion [36], [38], [54], [58].

255 Expansion anisotropy in materials subjected to loading is due to anisotropy of cracking. It
256 can be explained by the balance between the stresses induced by expansion around the
257 inclusion and the stresses due to external conditions. In stress free conditions, the
258 material is subjected to tensile stress around the expansive inclusions and the resulting
259 crack distribution is theoretically isotropic if the inclusions are isotropic and uniformly
260 distributed (Figure 3). Under compressive stresses, the tensile stresses are reduced (or
261 even cancelled by the compressive stresses due to the external loading) in the direction
262 parallel to the compressive stress and cracking is prevented in the direction perpendicular
263 to the applied load (Figure 3). The stress state is not modified along the direction
264 perpendicular to loading and cracking appears when, locally, the stress reaches the tensile
265 strength (Figure 3). This explains why the cracks are usually parallel to the compressed
266 direction or to the restraint whatever the nature of the new phases [30], [35], [38], [43].

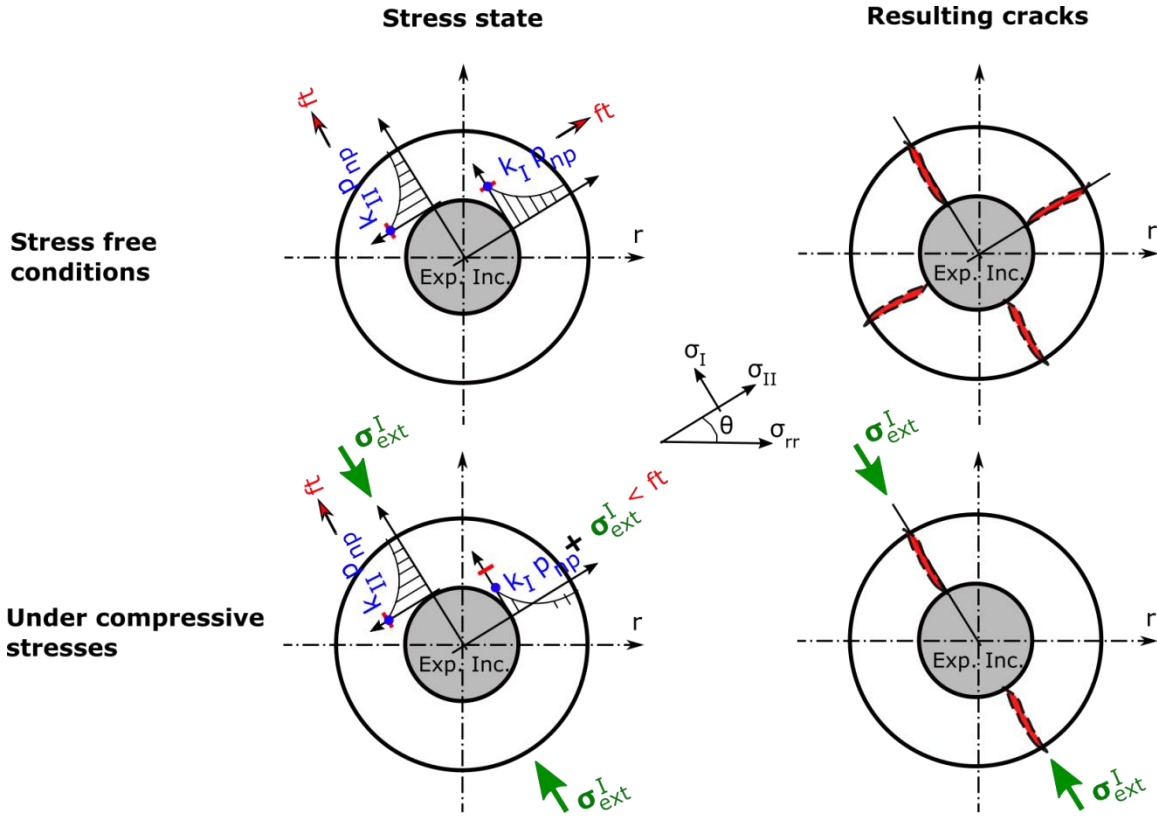


Figure 3: Idealized stress state around expansive inclusion, and resulting cracks. Due to the stress field shift towards compression, the failure criterion cannot be reached perpendicular to the applied compression, but only parallel to it

268 Once cracks occur in one direction, the following expansions change since the new
 269 phases can flow into or be formed in the cracks and pressure can be preferentially exerted
 270 perpendicular to the cracks. In cases of compression along two directions, cracking can
 271 occur parallel to the loading plane. In presence of loading, the anisotropic diffuse
 272 cracking can be evaluated through the following criteria f_i :

$$\begin{cases} f_I = k_I \cdot p_{np} + \tilde{\sigma}_I - f_t \\ f_{II} = k_{II} \cdot p_{np} + \tilde{\sigma}_{II} - f_t \\ f_{III} = k_{III} \cdot p_{np} + \tilde{\sigma}_{III} - f_t \end{cases} \quad (15)$$

273 with $\tilde{\sigma}_I$ the external stress in the material without damage (effective stress) in the main
 274 direction I of stresses (in this case, the stress concentration factors k_I are computed in the
 275 main direction of stresses).

276 For cracking to occur in stress-free conditions, the local tensile stress, equal to $k_I \cdot p_{np}$,
277 has to be greater than the local strength f_t (Equation (12)). In cases of compressive stress
278 induced by external loading, $\tilde{\sigma}_I$ is negative (Equation (15)). To lead to cracking, the local
279 stress induced by the pressure ($k_I \cdot p_{np}$) has to be higher than the sum of the tensile
280 strength and the absolute value of the compressive stress.

281 As the loading stress $\tilde{\sigma}_I$ is limited by the tensile strength f_t in the direction I [42], the
282 diffuse cracking depends directly on the pressure of the new phases for a material cracked
283 by external loading. The criterion becomes:

$$f_I = k_I \cdot p_{np} \quad (16)$$

284 If the external principal stresses do not coincide with the material main directions, the
285 stress concentration factors are determined in the principal directions of stresses
286 according to elliptic approximations. Thus, the cracks orientation is mainly driven by the
287 external principal stresses which is usually observed in real cases.

288 **3.3 Pressure evolution during diffuse cracking**

289 The volume of new phases changes over time: frost can disappear with increasing
290 temperature, alkali-silica gels can lose water when exposed to dry conditions, etc. After
291 such modifications, a large part of the expansion remains unchanged [5], [50], [59], so
292 the diffuse cracking induced by expansive phenomena does not appear to be reclosed.
293 This can be explained by the relative displacements of the lips of rough cracks. Lip
294 displacements lead to irreversible strains. This situation is modelled by means of the
295 plasticity theory that has already been used to represent permanent strain in quasi-brittle
296 materials [8], [9]. A plastic criterion is thus used to manage the permanent strain due to
297 the diffuse cracking induced by the pressure. It has already been used to model ettringite
298 formation [60] and alkali-silica reaction [61] expansions subjected to stress at macro-
299 scale. In these previous works, the anisotropy of cracking with loading (Figure 3) was
300 already considered but, in the present paper, the plastic criterion is modified to also
301 consider a possible initial anisotropy of the material through the inclusion shape.

302 The behaviour of quasi-brittle materials subjected to expansion after first cracking has to
303 be linked to experimental observations performed on specimens under restraint or applied
304 stresses:

- 305 - Under restraint, the expansive mechanism leads to compressive stresses at
306 macroscopic scale [30], [35]–[38]. In three experimental studies [30], [35], [36],
307 rigid steel frames were used to strongly restrained expansion in one direction (the
308 rigidity of the frames were largely upper than the rigidity of concrete specimens).
309 In these three cases, for concrete expansion due to ASR or ettringite formation,
310 the free swelling ranges from 0.1% to 8% and the resulting macroscopic uniaxial
311 stress due to restraint lies between only 1 and 8 MPa [30], [35], [36] (Figure 4).
312 The relation between free swelling and restraining stress appears to be quite linear
313 between 0 and about 0.4%, with an increase of 1 MPa per 0.1% of free expansion.
314 After 4 MPa, the increase of uniaxial stress with the free expansion slows down.
315 In comparison, for a material with a Young's modulus of 30,000 MPa, an
316 expansion of only 0.1% imposed on a perfectly elastic material would lead to
317 restraint stress of 30 MPa.
- 318 - Under applied loading, the application of uniaxial compressive stresses of about
319 5 MPa is usually sufficient to stop expansion of about 0.2% in the loaded
320 direction [43], [55]. Beyond this stress, the creep strains in the loaded direction
321 appear to be the same as for creep in the material without expansion [43], [54],
322 [55].

323 In these experiments, restraint and applied loading were uniaxial, and cracks could
324 propagate in plans perpendicular to free directions. The new phases can fill this cracking,
325 which delays the increase of the pressure in the porous materials and thus the resulting
326 stress. As a result, the restraint stress is not so high as evaluated with the elastic
327 assumption.

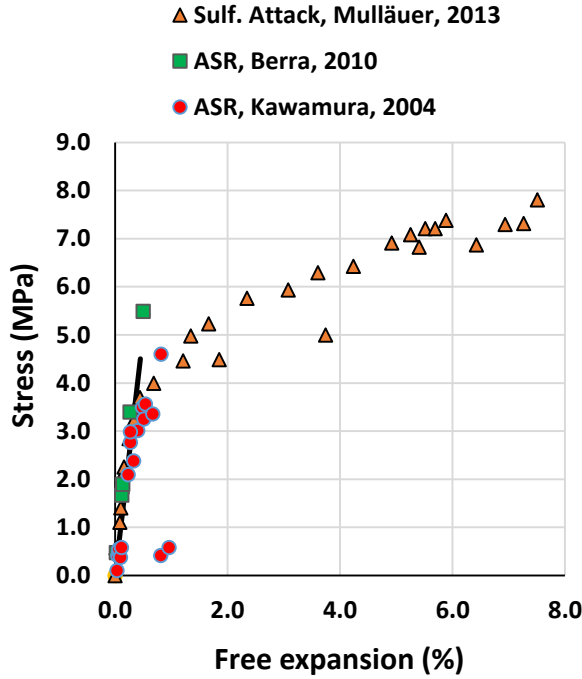


Figure 4: Restraint stress according to expansion in stress free condition for ASR-gels and ettringite in different strong restraint conditions [30], [35], [36].

328 This behaviour is taken into account through the combination of the pressure equation
 329 (Equation (1)) with the following hardening law (17). First, when cracking occurs, the
 330 volume of diffuse cracks φ_{cr} increases. The pressure p_i (Equation (1)) and the resulting
 331 macroscopic stress cannot increase too much. However, the first crack occurs in the
 332 weakest zone of the matrix. The tensile strength of the material after the first cracking
 333 increases continuously. The cracking process can continue only if the pressure grows in
 334 the zone that is not yet damaged. The stress has to increase in the undamaged zone (as
 335 observed in Figure 4) despite the fact that the specimen is free to swell in two directions
 336 (and restrained only in one direction). In this aim, a hardening law is proposed. This law
 337 has to be anisotropic as explained above:

$$\begin{cases} R_{tI} = f_t + H \cdot \varepsilon_{pl}^I \\ R_{tII} = f_t + H \cdot \varepsilon_{pl}^{II} \\ R_{tIII} = f_t + H \cdot \varepsilon_{pl}^{III} \end{cases} \quad (17)$$

338 with ε_{pl}^I , the plastic deformation in the main direction I. It represents the diffuse crack
339 opening in this direction. The sum of the plastic deformations in the three directions is
340 equal to φ_{cr} , the volume of diffuse cracks due to the pressure:

$$\varphi_{cr} = \varepsilon_{pl}^I + \varepsilon_{pl}^{II} + \varepsilon_{pl}^{III} \quad (18)$$

341 and

$$H = h_{crk} \cdot E \quad (19)$$

342 with h_{crk} the relative slope of the hardening law after cracking and E the elastic modulus
343 of the macro scale material before cracking.

344 Accurate evaluation of stress level for restraint expansions is obtained by the combination
345 of this hardening law with damage (Equation (11)) and creep modelling. The steeper the
346 slope of the hardening law after cracking, the higher the restraint stress obtained for a
347 given expansion. Figure 4 shows that the hardening law can be approximated by a linear
348 law for expansion lower than about 0.4%. The law should be nonlinear for higher
349 expansion (or at least bi-linear for the domain after cracking to consider the stiffness
350 decrease of the matrix). In this paper, h_{crk} is equal to 0.025 except when this parameter is
351 the object of a parametric study.

352 For triaxial compressive applied loading, pressures first have to counteract the
353 compressive stress before inducing tension and have to be still greater to cause cracks.
354 For such multiaxial loading, the experiments in the literature show that the measured
355 volumetric expansion decreases [56], [57]. In the model, as long as the cracking criterion
356 (Equation (15)) is not reached, the volume of cracking, φ_{cr} , stays equal to zero and the
357 pressure p_i increases (Equation (1)). As the pressure increases, φ_i^0 , the volumetric
358 fraction of new phases leading to small expansion also increases (Equation (2)) to
359 reproduce the dependence on pressure of the migration of phases in connected porosity
360 [56]. Thus, the pressure can never be too high even for triaxial compressive applied
361 loading [61] and the resulting volumetric expansion decreases.

362 The implementation of this model is based, first, on a classical implicit return mapping
363 algorithm [62] to solve the elasto-plastic problem, then the damage model [42] is applied

364 to the stresses deduced from the poro-plastic solution. The return mapping algorithm and
 365 the damage models are implemented in a non-linear finite element code. The method
 366 consists of first computing the evolution of the chemical reaction [63] versus the time
 367 step. Then, assuming visco-elastic behaviour of the poro-mechanical media, pressure and
 368 stress increments are computed according to the increment of strain. Once these
 369 increments have been added to the values of the last time step, the plastic criteria (15) are
 370 tested. If the criteria are exceeded, plastic flow is activated to return to the yield functions
 371 through the return mapping implicit method and the admissible stress state and pressure
 372 state obtained are used in the damage model to assess the apparent stress state. The finite
 373 element algorithm then computes the balance between the external forces applied to the
 374 structure and the internal forces corresponding to the stresses. If a force balance is not
 375 reached, the algorithm corrects the displacement field until equilibrium is reached.

376 4. Applications

377 The aim of this part is to evaluate the capability of the model to reproduce expansions in
 378 concrete in several situations. The validation is performed on concrete subjected to
 379 expansion due to alkali-silica reaction drawn from [50], [54]. In this study, the expansion
 380 occurred in reactive silica embedded in non-reactive limestone for two sizes of
 381 aggregates (Table 1). Thus, upscaling and calculations based on several pressures were
 382 used. The Young's moduli of the new phases, aggregate and concrete were taken to be
 383 7 GPa [64], [65], 60 GPa [43] and 37.3 GPa [50] respectively. The moduli of the different
 384 phases used in the calculations are given in Table 2. The concrete tensile and
 385 compressive strengths were 4 and 38.4 MPa [50]. The behaviour of concrete in creep is
 386 available in [54].

387 In this application, the advancement was evaluated through mass balance equations
 388 solved for each size of aggregates, taking account of alkali diffusion (Equation (20)) and
 389 reactive mechanisms deduced from alkali fixation by ASR-gels (Equation (21)) [63]:

$$\left\{ \begin{array}{l} p_{agg} S_r \frac{\partial [Na^+]}{\partial t} = div(D_a \overrightarrow{grad}[Na^+]) - \frac{\partial Na_f}{\partial t} \\ \frac{\partial Na_f}{\partial t} = \frac{\langle [Na^+] - [Na^+]_{(Ca,T)}^{thr} \rangle^+}{\tau_{ASR}} \end{array} \right. \quad (20)$$

$$\left\{ \begin{array}{l} p_{agg} S_r \frac{\partial [Na^+]}{\partial t} = div(D_a \overrightarrow{grad}[Na^+]) - \frac{\partial Na_f}{\partial t} \\ \frac{\partial Na_f}{\partial t} = \frac{\langle [Na^+] - [Na^+]_{(Ca,T)}^{thr} \rangle^+}{\tau_{ASR}} \end{array} \right. \quad (21)$$

390 with p_{agg} the aggregate porosity, S_r the degree of water saturation, $[Na^+]$ the alkali
 391 concentration in solution, Na_f , the amount of alkali bound in ASR-gels, D_a , the
 392 coefficient of diffusion of alkali in the aggregate, τ_{ASR} , the characteristic time of silica
 393 attack and $[Na^+]_{(Ca,T)}^{thr}$ the alkali concentration threshold below which the reaction
 394 products cause negligible expansion. This threshold depends on temperature T and
 395 calcium concentration, $[Ca^{2+}]$. In first approach, empirical relations can be used:

$$\left\{ \begin{array}{l} [Na^+]_{(Ca,T)}^{thr} = 1540 \cdot \exp \left[-\frac{205900}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_0} \right) \right] \cdot [Ca^{2+}] \\ [Ca^{2+}] = 0.357 \cdot \exp(386.8 \cdot [Na^+] - 0.01 \cdot T - 1.36 \cdot [Na^+] \cdot T) \end{array} \right. \quad (22)$$

$$\left\{ \begin{array}{l} [Na^+]_{(Ca,T)}^{thr} = 1540 \cdot \exp \left[-\frac{205900}{R} \cdot \left(\frac{1}{T} - \frac{1}{T_0} \right) \right] \cdot [Ca^{2+}] \\ [Ca^{2+}] = 0.357 \cdot \exp(386.8 \cdot [Na^+] - 0.01 \cdot T - 1.36 \cdot [Na^+] \cdot T) \end{array} \right. \quad (23)$$

396 With T in Kelvin and T_0 , equal to 293 K.

397 Usually, the molar ratio between silica (SiO_2) and alkali (Na_2O) present in ASR-gels in
 398 laboratory conditions is about 5 [64], [66]. The advancement of ASR-gel moles produced
 399 by one size of aggregate is then equal to the number of moles of silica attacked by alkalis
 400 divided by the maximal number of moles of the new products which can be formed in the
 401 aggregate:

$$A_a(t) = \frac{5 \partial Na_f}{2 \frac{\partial t}{n_i^{max}}} \quad (24)$$

402 The fraction of new phases created before cracking, φ_{ref}^a , per volume of aggregate a
 403 (Equation (2)) was evaluated from the surface area of the aggregates. In this work, it was
 404 taken to be:

$$\varphi_{ref}^a = 1 - \frac{(R_a - d_{ref})^3}{R_a^3} \quad (25)$$

405 with R_a the mean radius of the aggregate a and d_{ref} the distance used to evaluate the
 406 volume of new phases created before diffuse cracking. In reality, the volume of new
 407 phases formed before cracking is partly included in the aggregate porosity. For the sake
 408 of simplicity, only an average distance, d_{ref} , around the aggregate is considered. It is
 409 assumed not to depend on the aggregate size. The reference pressure (Equation (2)) is the
 410 pressure leading to the first diffuse cracking. p_{ref} depends on the tensile strength of the

411 porous material, f_t , and on aggregate shape through the stress concentration factor k_I
 412 (Figure 2):

$$p_{ref} = \frac{f_t}{k_I} \quad (26)$$

413

Table 1: Distribution of the aggregate embedded in the concrete studied [50], [54]

Aggregate reference (a)	Aggregate size (mm)	φ_a	b_a
1	4-12	0.10	0.19
2	12-20	0.32	0.49

414

Table 2: Mechanical properties of the different phases

Properties	New phases	Aggregate	Concrete
Young's modulus (GPa)	7	60	37.3
Poisson's ratio	0.2	0.2	0.2
Bulk modulus K (GPa)	3.9	33.3	20.7
Shear modulus μ (GPa)	2.9	25	15.5

415

416 **4.1 Anisotropy in stress-free conditions**

417 The model is first used to reproduce the anisotropic expansion due to a natural defect of
 418 the materials (Figure 2). In concrete, water pockets can form under aggregate particles,
 419 leading to stress concentrations (Figure 2) and thus to anisotropy [43], [53]. For the
 420 aggregate used in these experimentations, expansion was anisotropic in stress-free
 421 conditions: twice as large in the vertical direction as in the horizontal ones (circles in
 422 Figure 5). Isotropic expansion (calculated as the average expansion in the three directions

423 to obtain the same volumetric expansion – triangles in Figure 5) was first modelled with
424 the following parameters:

- 425 - the coefficient of diffusion of alkali in aggregate, D_a : $5 \cdot 10^{-14}$ m²/s,
- 426 - the characteristic time of silica attack, τ_{ASR} : 400 days,
- 427 - the molar volume of new phases, φ_{np}^{mol} : 15.5 cm³/mol,
- 428 - the average distance used to represent the volume of new phases created before
429 diffuse cracking, d_{ref} : 5 μ m.

430 The four parameters have been calibrated on the experimental data to reproduce the
431 isotropic expansions (Figure 5). They can be compared to experimental data of the
432 literature. The coefficient of diffusion of alkali in aggregate is smaller than the
433 coefficients measured in [67]. But such measurements are scarce, and the technique used
434 in this experimental work can be not totally representative of mechanisms occurring
435 during ASR-attack. The characteristic time of attack is slightly larger than the time
436 determined in [68] for opal, which can be expected due to the difference of silica nature
437 (opal is more reactive than aggregate used in this application). Molar volume of synthetic
438 gels similar to natural ASR gels in solution was measured to be between 17 and 24
439 cm³/mol depending on the composition of the gels [69]. The calibration is thus in good
440 agreement with these experimental values obtained on synthetic gels. As d_{ref} is an
441 average distance, it cannot be directly compared to microscopic observations. However, it
442 can be supposed to be realistic as it is lower than usual interfacial transition zone between
443 aggregate and cement paste which can reach 20 μ m [70], [71].

444 φ_{np_a} , the volume of new phases of the aggregate a at the time step t (Equation (7)), is
445 proportional to the molar volume of new phases φ_{np}^{mol} according to the advancement of
446 the reaction (Equation (6)). In this calculation, it leads to final volumes of gel φ_{np_1} and
447 φ_{np_2} respectively equal to $1.9 \cdot 10^{-2}$ and $0.9 \cdot 10^{-2}$ m³ of new phases per m³ of aggregate
448 (Table 3). As the kinetics is partly driven by ionic diffusion in spherical particles, the
449 volume of new phases is the largest in the smallest reactive aggregate. The Biot
450 coefficients and moduli computed for the new phases are given in Table 3. The Biot
451 coefficient of the new phases in the smallest aggregate is larger than that in the largest

452 aggregate because of differences of volume of the phases. In this experiment, not all the
453 reactive silica was consumed at the end of expansion. If all the silica had been consumed
454 in all the aggregates, the same volume of new phases would have been created per
455 volume of aggregate and thus the Biot coefficients would have been equal for the two
456 aggregates.

457 The rate of anisotropy, ρ_{ani} , is then calibrated (and taken equal to 0.83) to obtain the
458 anisotropic expansion (Figure 5). The criterion proposed in Equations (15) and (16) used
459 with Equation (14) to obtain anisotropic cracking is then effective to reproduce the
460 anisotropy of expansion in stress-free conditions with the same parameters (φ_{np}^{mol} and
461 d_{ref}) as the averaged isotropic expansion. The formulation of the plasticity law for
462 materials with high anisotropy of expansion in stress-conditions is thus validated. This
463 feature of the model should be also supported by the comparison with expansion of the
464 same material submitted to other stress levels. But experimental data with high
465 anisotropic expansion are scarce and no experimental program have supplied data under
466 various stress state for such materials.

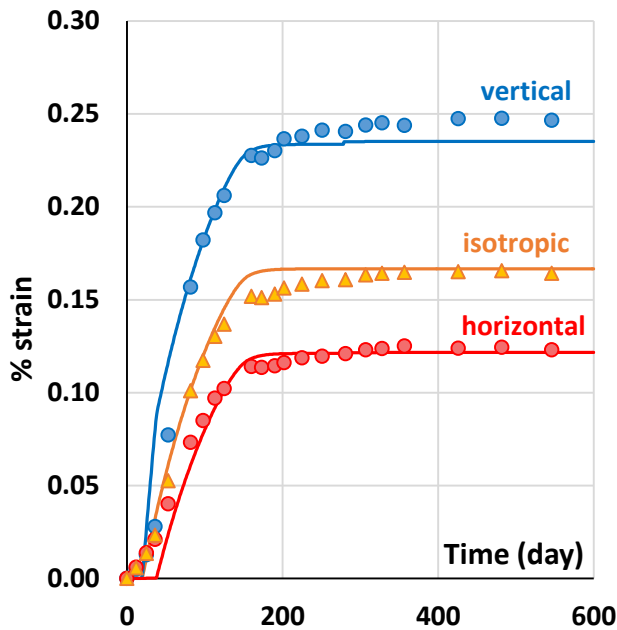


Figure 5: Isotropic or anisotropic stress-free expansion (circles: experimental points [50], triangles: averaged isotropic expansion deduced from experimental points, lines: modelling)

Table 3: Final volume of new phases and Biot coefficient and modulus for each aggregate class

Aggregate size	mm	4-12	12-20
φ_{np_a}	-	1.9e-2	0.9e-2
b_{np}	-	3.8e-2	1.8e-2
M_{np}	GPa	199.	429.

467

468 **4.2 Expansion under restraint**

469 The dependence of expansion on stress has to be well reproduced in order to obtain
470 relevant modelling of real cases in specimens subjected to gradients or in real structures
471 subjected to external loading or restraint due to boundary conditions. In the macro
472 modelling proposed in [11], [61], which is efficient to reproduce structural behaviour, the
473 Biot modulus and coefficient were used as calibration parameters to reproduce the
474 behaviour of concrete subject to expansion and under mechanical loading and restraint. In
475 the present work, the Biot modulus and coefficient are determined from the volume of
476 new phases and from the mechanical properties of concrete, aggregate and new phases. In
477 this case, only the slope of the hardening law after cracking h_{crk} in equation (18) is
478 unknown. This parameter allows the pressure growth to be determined during the
479 cracking process needed to explain the longitudinal stress increase observed in the
480 experiments reported in Figure 4. The present paper proposes a parametric study to verify
481 the capability of the model to reproduce the dependence of expansion on stress with only
482 the slope of the hardening law as calibration parameter.

483 The impact of h_{crk} on restrained stress is studied first. Three sets of parameters (Table 4)
484 are determined for three significantly different values to obtain the same stress-free
485 expansion (about 0.17% of isotropic linear expansion in Figure 5). The greater the slope
486 of the hardening law, h_{crk} , the greater the pressure increase and the axial stress after
487 cracking. In consequence, the molar volume φ_{np}^{mol} necessary to obtain the same stress-
488 free expansion has to be larger for a greater h_{crk} . This is due to the fact that, when

489 pressure growth is possible (when h_{crk} is great enough), the amount of gel migrating to
 490 the connected porosity also increases (see Equation (2)), so the amount of gel remaining
 491 in the inclusion to maintain the pressure would be smaller if the molar volume was not
 492 increased artificially to retrieve the correct free swelling.
 493

Table 4: Modelling parameters for the study of restraint stress

h_{crk}	φ_{np}^{mol}	d_{ref}
-	(cm ³ /mol)	(μ m)
0.002	14.	4.5
0.025	15.5	5.
0.2	27.5	6.

494
 495 Expansion under uniaxial restraint is then modelled. In this aim, the strain has been set to
 496 zero in the restraint direction. The numerical restraint stresses are plotted in Figure 6 for
 497 the three values of slope: the greater the slope, the larger the restraint stress. For h_{crk}
 498 equal to 0.002, the restraint stress is close to zero. This means that the pressure remains
 499 almost constant during crack propagation, as can happen for very fluid new phases
 500 (Figure 4 and Figure 7). In most cases, the restraint stress is about 1 MPa for 0.1%
 501 expansion and up to 0.4 or 0.5% expansion (Figure 7). A correct mean representation is
 502 obtained for a slope of about 0.025 (restraint stress of about 1.65 MPa for a final isotropic
 503 expansion of about 0.17%, Figure 7). For h_{crk} equal to 0.2, the calculation is closer to the
 504 elastic calculation than in the previous cases and restraint stress becomes too high (Figure
 505 6 and Figure 7). The end of the production of new phases occurs at about 180 days and
 506 free expansion stabilizes by this date (Figure 5). For restraint expansion, after the
 507 stabilization of expansion, a decrease of the restraint stress can be observed (Figure 6). It
 508 is due to the concrete relaxation, which is caused by the concrete viscoelastic behaviour,
 509 here, fitted on creep tests [41].

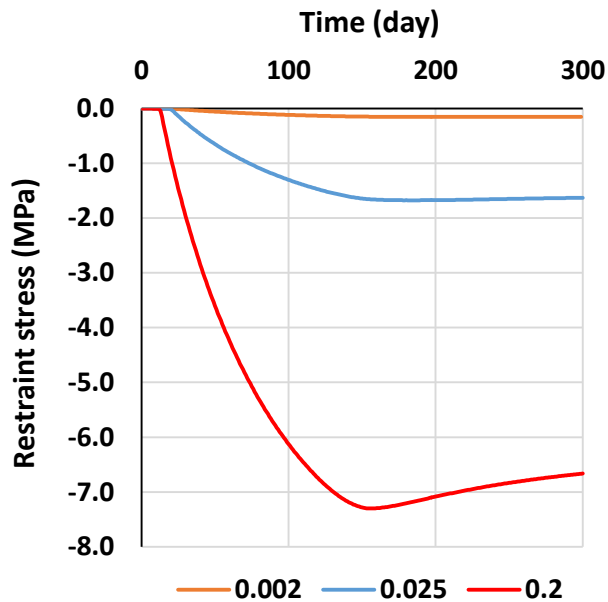


Figure 6: Restraint stress for an expansion of about 0.16% according to h_{crk}

510

511 In this paper, the calibration of the parameter h_{crk} has been performed to obtain a
 512 restraint stress of about 1 MPa for an expansion in stress-free conditions of about 0.1%
 513 (h_{crk} equal to 0.025). It is a usual mean value for expansive mechanisms under restraint.
 514 The modelling validation is performed in Figure 7 with the comparison with several
 515 experimental studies [30], [35], [36]. These works were performed for different initial
 516 materials and different new phases (ASR or ettringite). The modelling is able to evaluate
 517 the stress for restrained expansion of the three experimental studies, if expansion in
 518 stress-free condition is lower than 0.4%. A non-linear hardening law could give better
 519 results for larger expansions but the calculations would be more time-consuming (in
 520 cases of linear hardening, the implicit return mapping algorithm converges in one step
 521 only, which is no longer the case for nonlinear hardening). For phenomena with greater
 522 expansion (delayed ettringite formation, sulfate attack), a multi-linear description could
 523 lead to a more realistic evaluation without consuming too much computational time.

524 In some experiments, very small stress was noted for material with large potential for
 525 expansion in stress free conditions (Figure 7, [35]). It was due to exudation of products
 526 out of the specimens as observed by the authors of [35]. Such phenomena are not intrinsic

527 to the material and should not be reproduced by material modelling but by coupled
 528 modelling as proposed in [72]. Future works will combine the present modelling with
 529 transport modelling to reproduce such mechanisms.

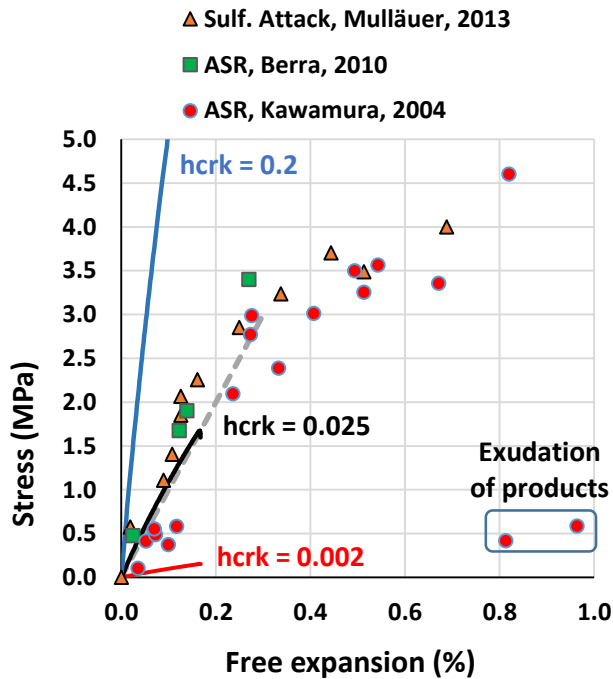


Figure 7: Restraint stress according to expansion in stress free condition, comparison with modelling according to h_{crk} (from 0.002 to 0.2), experimental data from [30], [35], [36]

530 **4.3 Irreversibility of expansion during drying**

531 The aim of this part is to analyse the ability of the model to reproduce the behaviour of
 532 concrete undergoing first expansion then drying shrinkage. After 650 days of expansion
 533 for the concrete analysed in part 3.1, specimens were exposed to air at 30% RH [50].
 534 When the new phases due to alkali-silica reaction dried, their volume decreased because
 535 of water loss, but the concrete expansion did not change; only the usual concrete drying
 536 shrinkage occurred (Figure 8, [50]).

537 For the calculations below, the same parameters as in part 3.1 are used for expansion.
 538 Drying shrinkage is obtained through the Van Genuchten model [73] as explained in [41].
 539 In this case, Equation (9) becomes:

$$\tilde{\sigma}_s = K_d \cdot (\varepsilon - \varepsilon_{an}) - \sum_a b_a \cdot p_a - b_w \cdot p_w \quad (27)$$

540 with b_w , the Biot coefficient of water (dependent on the saturation degree of the porosity
 541 in the material). p_w is the pressure due to water, which is negative in case of shrinkage
 542 (due the capillarity effect) and zero during swelling in water. The pressure p_w is
 543 modelled using the Van Genuchten model [73]:

$$p_w = -M_{shr} \cdot \left(S_r^{\frac{1}{m_{vg}}} - 1 \right)^{(1-m_{vg})} \quad (28)$$

544 M_{shr} and m_{vg} can be calibrated on the water retention curve or on the evolution of
 545 shrinkage strains.

546

547 In the present work, the Biot coefficient of water b_w has been evaluated from the
 548 concrete porosity (16%) [74]. It is equal to 0.4 for this concrete in case of total saturation.
 549 m_{vg} has been taken equal to the usual value 0.5, and only M_{shr} has to be calibrated. M_{shr}
 550 is evaluated to reproduce the shrinkage strain on a concrete which has never been
 551 damaged by ASR (Figure 8-a). It is equal to 32 MPa, which is a usual value for concrete
 552 [41]. The validation of the model is performed on the same concrete which is submitted
 553 to the same drying but after ASR expansion (Figure 8-b). After expansion occurring in
 554 water, concrete shrinks with the usual dependence on water pressure (Figure 8-b).
 555 Cracking induced by the formation of the new phases did not change the amplitude of
 556 shrinkage significantly since the experimental results could be found without changing
 557 the coefficients of Equations (27) and (28) after swelling. The decrease of ASR-gel
 558 volumes induced by the drying did not impact the irreversible expansion: they were either
 559 filled by crystallized phases or were impossible to reclose simply because of the
 560 roughness of their lips. Figure 8-b shows that the behaviour observed on specimens can
 561 be reproduced by a basic combination of poromechanics (Equation (27)) with the linear
 562 anisotropic plasticity used in the constitutive equations.

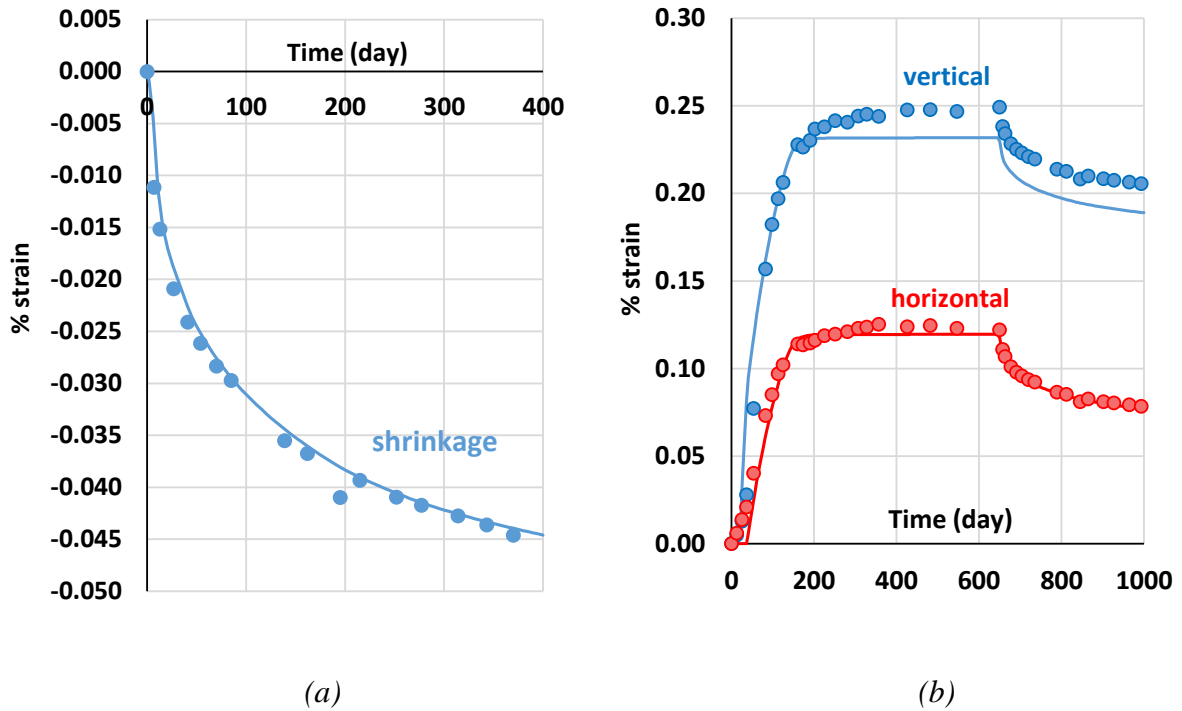


Figure 8: Calibration of shrinkage parameters on concrete which has never been damaged by ASR (a) and validation on the same concrete which is submitted to the same drying, after ASR expansion (b) (circles: experimental points [50], lines: modelling)

563 4.4 Damage due to pressure gradient

564 In the previous analysis, homogeneous conditions were assumed so as to analyse the
 565 response of the model from a material point of view without disturbance due to gradient
 566 effects. In reality, the specimens analysed just above were kept in water. Such storage
 567 leads to a pressure gradient due to alkali leaching: close to the specimen skin, alkalis
 568 leach out rapidly and cannot participate in the chemical reaction. As a result, chemical
 569 advancement is slower close to the skin than in specimen cores, which leads to pressure
 570 gradients and skin cracking. In the calculations, the risk of skin cracking and the cracks
 571 opening is evaluated by the model presented in [42]. The model is regularized with
 572 Hillerborg anisotropic method to avoid mesh dependence [75]. The calibration of the
 573 expansion curves in the two directions (Figure 9) is obtained for a molar volume of new
 574 phases, φ_{np}^{mol} , equal to 22.1 cm³/mol, an average distance over which the new phases can

575 move without causing cracking, d_{ref} , equal to 7 μm , and the anisotropy ratio, ρ_{ani} , equal
576 to 0.82. The radial deformation is the radial displacement in P1 divided by the specimen
577 radius, and the vertical deformation is the vertical displacement in P2 divided by the
578 measurement basis (five centimetres above the symmetry axis as during experimentation
579 – Figure 10). As a fraction of the reactants does not participate in the attack because of
580 their leaching, the molar volume has to be slightly higher than in the previous
581 calculations to obtain the same expansion. The gradient of advancement due to alkali
582 leaching leads to the pressure gradient presented in Figure 10-a. Thus, tensile stresses at
583 the external surface of concrete specimens (Figure 10-b) are sufficient to cause localized
584 cracking with openings larger than 100 μm (Figure 11), which is the cracking often
585 observed on specimens in laboratory and real structures. Such a gradient can be induced
586 by leaching or drying according to the environmental conditions (open cracks visible
587 without microscopic tools, Figure 12) [1]. The diffuse cracking due to pressure is usually
588 finer and forms a denser network (one or two cracks per aggregate [48]) and can also be
589 observed on real structures sometimes (Figure 12).
590

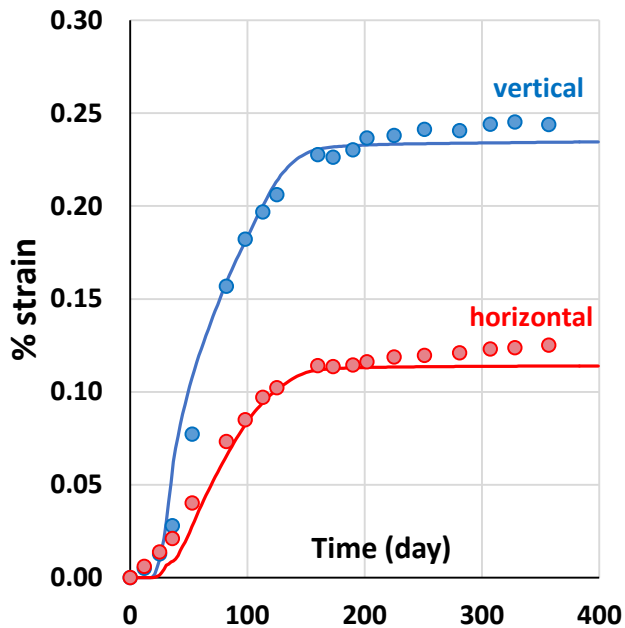


Figure 9: Comparison of calculated expansion and experimental data (circles: experimental points [50])

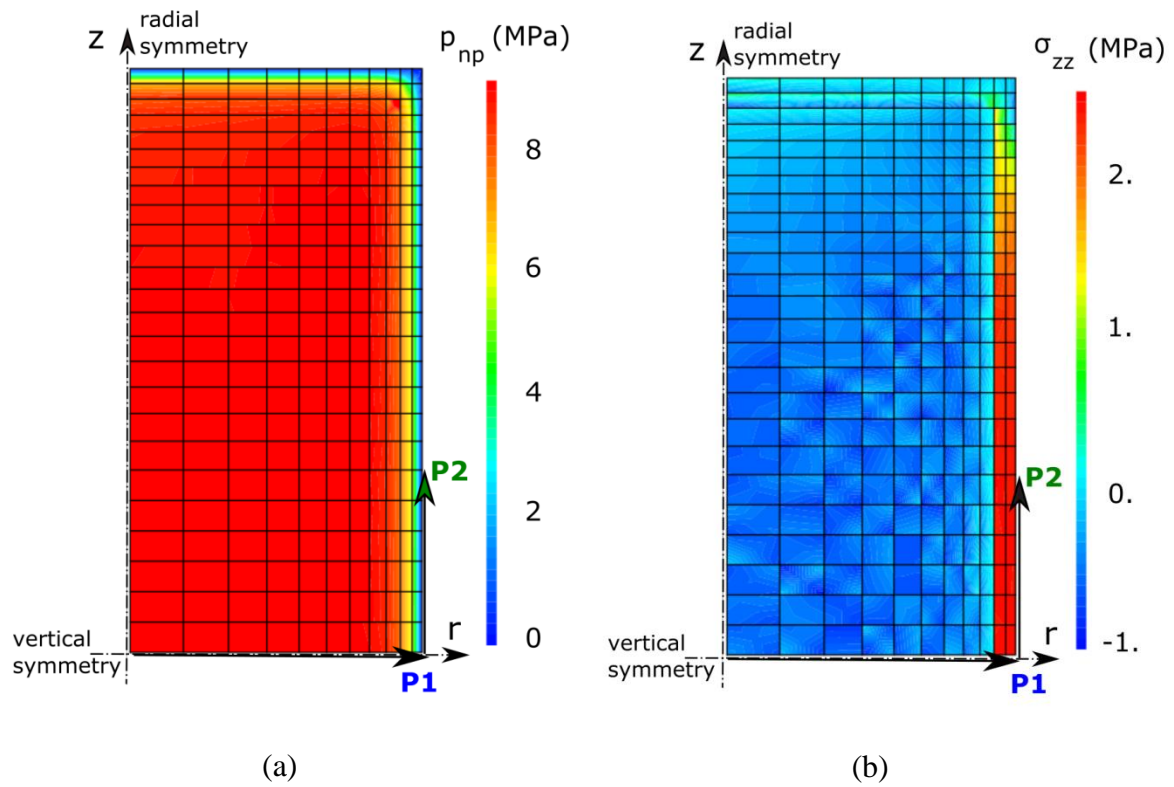


Figure 10: Pressure gradient due to alkali leaching (a) and induced stresses (b) just before skin cracking (in MPa)

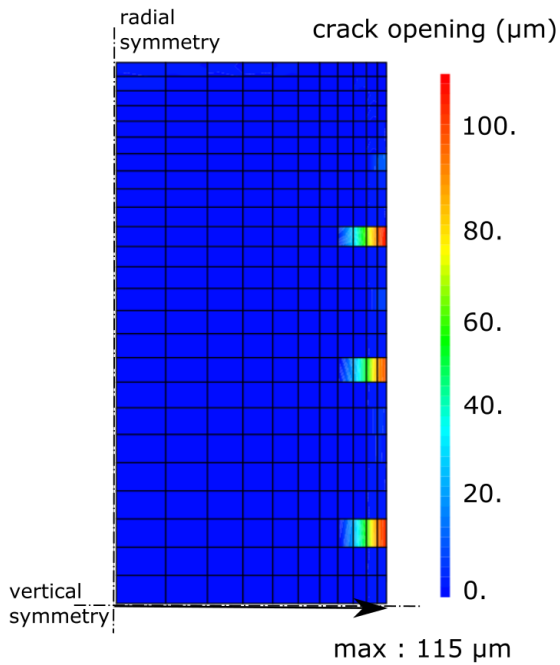


Figure 11: Cracking due to pressure gradient at the end of the test

591

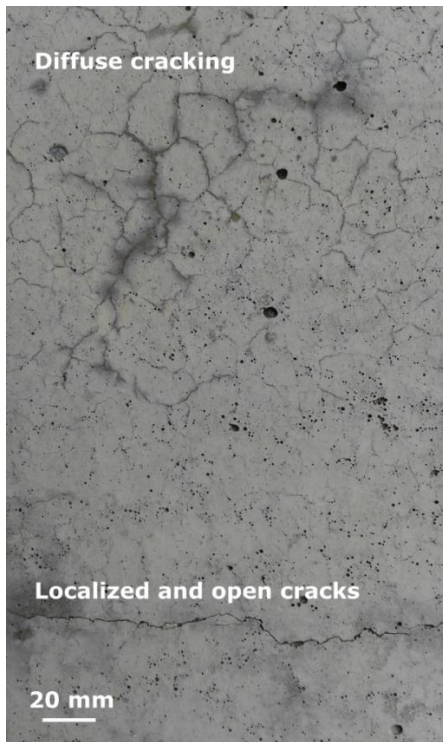


Figure 12: Diffuse and localized cracks on column damaged by expansive phenomena

592

593 Investigations showed that expansive materials were usually subjected to a pressure
594 gradient due to environmental conditions, even in laboratory tests performed on small
595 specimens. Gradients of temperature, moisture or ionic species lead to gradients of
596 pressure. Such gradients can lead to localized open cracks on the skin or in the core of
597 specimens. This strongly impacts the expansion measurements and the consequences in
598 terms of material damage evaluated on specimens. Diffuse and localized cracking do not
599 have the same impact in terms of strength and rigidity decreases and of transfer
600 properties. In the future, models will have to assess the crack opening due to pressure
601 gradients in real structures as it will impact the durability of the structure through the
602 induced modifications of transfer properties. Such an assessment can be performed from
603 crack openings for localized cracks or for diffuse cracking. Therefore, modelling of
604 expansive phenomena should be able to distinguish between diffuse damage due to
605 pressure and localized damage due to gradient. In this work, the distinction is achieved by
606 combining the plasticity modelling, used to represent the permanent strain due to diffuse

607 cracking induced by pressure presented above, with the damage modelling presented in
608 [42].

609 **4.5. Discussion**

610 The previous calculations were performed in the framework of poromechanics. The Biot
611 coefficient and modulus depend on the mechanical properties of the materials and on the
612 volume of porosity filled by the new phases. For the multi-scale modelling used to assess
613 the impact of alkali-silica reaction, the Biot coefficients of new phases in aggregate,
614 evaluated through Equation (5), lie between $3.8e-2$ (in homogeneous conditions) and
615 $5.4e-2$ (in the calculations with gradient) in the smallest aggregate (4/12) and between
616 $1.8e-2$ and $2.5e-2$ in the largest one (12/20) at the end of the calculated expansion. These
617 values are consistent with previous calculations performed for other new phases formed
618 in concrete [12], [13]. The Biot coefficients of aggregate in concrete reach about 0.2 in
619 the smaller aggregate (4/12) and 0.5 in the larger aggregates (4/12). This is consistent
620 with modelling that evaluates the expansion from the action of the aggregate on the
621 concrete [26], [61], [76] without representation of aggregate damage. It is usually
622 performed in structural calculations. The modelling proposed in this paper makes the link
623 between the two scales. Taking the shape of cracks into consideration could lead to
624 slightly larger coefficients [26], [40]. This means that the present modelling could
625 underestimate the percentage of expansion due to pressure and thus slightly overestimate
626 the percentage due to cracking. Pressure due to the formation of new phases and
627 evaluated by the present modelling lies between 10 and 15 MPa. These values are
628 consistent with usual values measured for such new phases [77], [78] or evaluated
629 through theoretical approaches [79], [80]. It is also a validation of the mechanical
630 developments proposed in this paper.

631

632 **5. Conclusion**

633 The main consequences of expansion in quasi-brittle materials are: anisotropic induced
634 cracking due to natural causes or due to external loading, stress for restrained expansion,
635 irreversibility of diffuse cracking and localized cracks due to pressure gradients.

636 A model of expansion in quasi-brittle materials has been proposed. It is based on
637 poromechanics and on a plastic, anisotropic criterion to represent permanent strain due to
638 diffuse cracking caused by pressure. Thus the anisotropy of cracking is induced by the
639 addition of local stress due to pressure and external loading. Equations have been
640 proposed for multiple pressures to take account of the effects of differences of
641 advancement in the formation of new products on pressure and induced damage.

642 The following physical phenomena can be reproduced by the model:

- 643 - Anisotropy of expansion and cracking due to the initial anisotropy properties of
644 the material,
- 645 - Induced stress in conditions of restraint,
- 646 - Irreversibility of expansion during drying periods,
- 647 - Shrinkage of cracked drying concrete previously affected by expansion,
- 648 - Combination of diffuse damage due to expansion and localized damage due to
649 gradient.

650 The importance of diffuse cracking in the mechanisms of expansion in concrete has been
651 highlighted. This mechanical behaviour is strongly dependent on the mechanical
652 properties of the material subjected to expansion and particularly to properties in tension:
653 first for the diffuse damage and the direction of cracking due to pressure and, second, for
654 the localized cracks due to pressure gradient. The latter point introduces the
655 quantification of localized crack opening to evaluate the consequences of damage due to
656 a skin gradient in terms of mechanical and transfer properties. Cracks are usually
657 observed along reinforcements in structures subjected to expansion. These cracks shorten
658 the service life of damaged structures and the reduction can be evaluated from the crack
659 opening. This is an important issue when modelling the durability of structures affected
660 by expansion.

662 **6. References**

- 663 [1] B. Fournier and M.-A. Bérubé, “Alkali-aggregate reaction in concrete : A review
664 of basic concepts and engineering implications,” *Can. J. Civ. Eng.*, vol. 27, pp.
665 167–191, 2000.
- 666 [2] S. Diamond, “Delayed ettringite formation - Processes and problems,” *Cem.*
667 *Concr. Compos.*, vol. 18, pp. 205–215, 1996.
- 668 [3] C. Ouyang, A. Nanni, and W. F. Chang, “Internal and external sources of sulfate
669 ions in Portland cement mortar; two types of chemical attack,” *Cem. Concr. Res.*,
670 vol. 18, pp. 699–709, 1988.
- 671 [4] M. Neji, B. Bary, P. Le Bescop, and N. Burlion, “Swelling behavior of ion
672 exchange resins incorporated in tri-calcium silicate cement matrix: I. Chemical
673 analysis,” *J. Nucl. Mater.*, vol. 467, pp. 544–556, 2015.
- 674 [5] C. Huang and R. Feldman, “Dependence of Frost Resistance on the Pore Structure
675 of Mortar Containing Silica Fume,” *ACI J.*, vol. 82, no. 5, pp. 740–743, 1985.
- 676 [6] O. Coussy, *Poromechanics*, Wiley. New York, 2004.
- 677 [7] L. Dormieux, D. Kondo, and F. Ulm, *Microporomechanics*, Wiley. 2006.
- 678 [8] J. Lubliner, J. Oliver, S. Oller, and E. Onate, “A plastic-damage model for
679 concrete,” *Int. J. Solids Struct.*, vol. 25, no. 3, pp. 299–326, 1989.
- 680 [9] A. D. Jefferson, “Craft — a plastic-damage-contact model for concrete . I . Model
681 theory and thermodynamic considerations,” *Int. J. Solids Struct.*, vol. 40, pp.
682 5973–5999, 2003.
- 683 [10] B. Capra and A. Sellier, “Orthotropic modelling of alkali-aggregate reaction in
684 concrete structures : numerical simulations,” *Mech. Mater.*, vol. 6, no. December,
685 2002.
- 686 [11] E. Grimal, A. Sellier, Y. Le Pape, and E. Bourdarot, “Creep, Shrinkage, and
687 Anisotropic Damage in Alkali-Aggregate Reaction Swelling Mechanism-Part I. A
688 Constitutive Model,” *ACI Mater. J.*, vol. 105, no. 3, pp. 227–235, 2008.
- 689 [12] B. Bary, N. Leterrier, E. Deville, and P. Le Bescop, “Coupled chemo-transport-
690 mechanical modelling and numerical simulation of external sulfate attack in
691 mortar,” *Cem. Concr. Compos.*, vol. 49, pp. 70–83, 2014.
- 692 [13] B. Bary, “Simplified coupled chemo-mechanical modeling of cement pastes
693 behavior subjected to combined leaching and external sulfate attack,” *Int. J.*
694 *Numer. Anal. Methods Geomech.*, vol. 32, no. February, pp. 1791–1816, 2008.
- 695 [14] R. J. Flatt and G. W. Scherer, “Thermodynamics of crystallization stresses in
696 DEF,” *Cem. Concr. Res.*, vol. 38, pp. 325–336, 2008.
- 697 [15] R. Tixier and B. Mobasher, “Modeling of Damage in Cement-Based Materials
698 Subjected to External Sulfate Attack . I : Formulation,” *ASCE J. Mater. Civ. Eng.*,

- 699 vol. 15, no. August, pp. 305–313, 2003.
- 700 [16] A. E. Idiart, C. M. López, and I. Carol, “Chemo-mechanical analysis of concrete
701 cracking and degradation due to external sulfate attack: A meso-scale model,”
702 *Cem. Concr. Compos.*, vol. 33, pp. 411–423, 2011.
- 703 [17] S. Sarkar, S. Mahadevan, J. C. L. Meeussen, H. Van Der Sloot, and D. S. Kosson,
704 “Numerical simulation of cementitious materials degradation under external
705 sulfate attack,” *Cem. Concr. Compos.*, vol. 32, no. 3, pp. 241–252, 2010.
- 706 [18] N. Cefis and C. Comi, “Chemo-mechanical modelling of the external sulfate attack
707 in concrete,” *Cem. Concr. Res.*, vol. 93, pp. 57–70, 2017.
- 708 [19] A. Gholizadeh, F. Rajabipour, and J. L. Rosenberger, “Composition – rheology
709 relationships in alkali – silica reaction gels and the impact on the gel’s deleterious
710 behavior,” *Cem. Concr. Res.*, vol. 83, pp. 45–56, 2016.
- 711 [20] W. Puatatsananon and V. Saouma, “Chemo-mechanical micromodel for alkali-
712 silica reaction,” *ACI Mater. J.*, vol. 110, no. 110, pp. 67–77, 2013.
- 713 [21] M. Alnagar, G. Cusatis, and G. Di Luzio, “Lattice Discrete Particle Modeling
714 (LDPM) of Alkali Silica Reaction (ASR) deterioration of concrete structures,”
715 *Cem. Concr. Compos.*, vol. 41, pp. 45–59, 2013.
- 716 [22] G. W. Scherer, “Crystallization in pores,” *Cem. Concr. Res.*, vol. 29, no.
717 December 1998, pp. 1347–1358, 1999.
- 718 [23] O. Coussy, “Deformation and stress from in-pore drying-induced crystallization of
719 salt,” *J. Mech. Phys. Solids*, vol. 54, pp. 1517–1547, 2006.
- 720 [24] O. Coussy, “Poromechanics of freezing materials,” *J. Mech. Phys. Solids*, vol. 53,
721 pp. 1689–1718, 2005.
- 722 [25] Q. Zeng, T. Fen-Chong, P. Dangla, and K. Li, “A study of freezing behavior of
723 cementitious materials by poromechanical approach,” *Int. J. Solids Struct.*, vol. 48,
724 no. 22–23, pp. 3267–3273, 2011.
- 725 [26] L. Charpin and A. Ehrlacher, “Microporomechanics study of anisotropy of ASR
726 under loading,” *Cem. Concr. Res.*, vol. 63, pp. 143–157, 2014.
- 727 [27] A. Sellier, J. P. Bournazel, and A. Mébarki, “Une modélisation de la réaction
728 alcalis-granulat intégrant une description des phénomènes aléatoires locaux,”
729 *Mater. Struct.*, vol. 28, no. 1, pp. 373–383, 1995.
- 730 [28] L. Charpin and A. Ehrlacher, “Simplified model for the transport of Alkali-Silica
731 Reaction gel in concrete porosity,” *J. Adv. Concr. Technol.*, vol. 12, pp. 1–6, 2014.
- 732 [29] Z. Bažant and S. Rahimi-Aghdam, “Diffusion-Controlled and Creep-Mitigated
733 ASR Damage via Microplane Model . I: Mass Concrete,” *J. Eng. Mech.*, vol. 143,
734 no. 3, pp. 1–10, 2017.
- 735 [30] W. Müllauer, R. E. Beddoe, and D. Heinz, “Sulfate attack expansion
736 mechanisms,” *Cem. Concr. Res.*, vol. 52, pp. 208–215, 2013.
- 737 [31] T. Ichikawa, “Alkali-silica reaction, pessimum effects and pozzolanic effect,”
738 *Cem. Concr. Res.*, vol. 39, pp. 716–726, 2009.

- 739 [32] T. Ichikawa and M. Miura, “Modified model of alkali-silica reaction,” *Cem.*
740 *Concr. Res.*, vol. 37, pp. 1291–1297, 2007.
- 741 [33] E. Garcia-Diaz, J. Riche, D. Bulteel, and C. Vernet, “Mechanism of damage for
742 the alkali-silica reaction,” *Cem. Concr. Res.*, vol. 36, pp. 395–400, 2006.
- 743 [34] E. Garcia-Diaz, D. Bulteel, Y. Monnin, P. Degrugilliers, and P. Fasseu, “ASR
744 pessimum behaviour of siliceous limestone aggregates,” *Cem. Concr. Res.*, vol. 40,
745 no. 4, pp. 546–549, 2010.
- 746 [35] M. Kawamura and K. Iwahori, “ASR gel composition and expansive pressure in
747 mortars under restraint,” *Cem. Concr. Compos.*, vol. 26, pp. 47–56, 2004.
- 748 [36] M. Berra, G. Faggiani, T. Mangialardi, and a. E. Paolini, “Influence of stress
749 restraint on the expansive behaviour of concrete affected by alkali-silica reaction,”
750 *Cem. Concr. Res.*, vol. 40, pp. 1403–1409, 2010.
- 751 [37] H. Kagimoto, Y. Yasuda, and M. Kawamura, “ASR expansion, expansive pressure
752 and cracking in concrete prisms under various degrees of restraint,” *Cem. Concr.*
753 *Res.*, vol. 59, pp. 1–15, 2014.
- 754 [38] H. Bouzabata, S. Multon, A. Sellier, and H. Houari, “Effects of restraint on
755 expansion due to delayed ettringite formation,” *Cem. Concr. Res.*, vol. 42, no. 7,
756 pp. 1024–1031, 2012.
- 757 [39] K. Mori and K. Tanaka, “Average stress in matrix and average elastic energy of
758 materials with misfitting inclusions,” *Acta Metallurg.*, vol. 21, no. 5, pp. 1605–
759 1609, 1973.
- 760 [40] R. Esposito and M. A. N. Hendriks, “A multiscale micromechanical approach to
761 model the deteriorating impact of alkali-silica reaction on concrete,” *Cem. Concr.*
762 *Compos.*, vol. 70, pp. 139–152, 2016.
- 763 [41] A. Sellier, S. Multon, L. Buffo-Lacarrière, T. Vidal, X. Bourbon, and G. Camps,
764 “Concrete creep modelling for structural applications: non-linearity, multi-axiality,
765 hydration, temperature and drying effects,” *Cem. Concr. Res.*, vol. 79, pp. 301–
766 315, 2016.
- 767 [42] A. Sellier, G. Casaux-Ginestet, L. Buffo-Lacarrière, and X. Bourbon, “Orthotropic
768 Damage Coupled with Localised Crack Reclosure Processing. Part I: Constitutive
769 Laws,” *Eng. Fract. Mech.*, vol. 971, no. January, pp. 148–167, Oct. 2013.
- 770 [43] C. Larive, “Apports combinés de l’expérimentation et de la modélisation à la
771 compréhension de l’alcali-réaction et de ses effets mécaniques, Laboratoire Central
772 des Ponts et Chaussées (Edt.), Ouvrage d’Art, Rapport OA 28,” 1998.
- 773 [44] F.-J. Ulm, O. Coussy, K. Li, and C. Larive, “Thermo-Chemo-Mechanics of ASR
774 expansion in concrete structures,” *ASCE J. Eng. Mech.*, vol. 126, no. March, pp.
775 233–242, 2000.
- 776 [45] J. Berard, R. Roux, I. Depuis, and C. Climinces, “La viabilite des betons du
777 Quebec: le role des granulats,” *Can J Civ Eng*, vol. 13, no. 1, pp. 12–24, 1986.
- 778 [46] J. M. Ponce and O. R. Batic, “Different manifestations of the alkali-silica reaction

- 779 in concrete according to the reaction kinetics of the reactive aggregate,” *Cem.*
780 *Concr. Res.*, vol. 36, pp. 1148–1156, 2006.
- 781 [47] M. Ben Haha, E. Gallucci, A. Guidoum, and K. L. Scrivener, “Relation of
782 expansion due to alkali silica reaction to the degree of reaction measured by SEM
783 image analysis,” *Cem. Concr. Res.*, vol. 37, pp. 1206–1214, 2007.
- 784 [48] L. F. M. Sanchez, B. Fournier, M. Jolin, and J. Duchesne, “Reliable quantification
785 of AAR damage through assessment of the Damage Rating Index (DRI),” *Cem.*
786 *Concr. Res.*, vol. 67, pp. 74–92, 2015.
- 787 [49] A. B. Giorla, K. L. Scrivener, and C. F. Dunant, “Influence of visco-elasticity on
788 the stress development induced by alkali–silica reaction,” *Cem. Concr. Res.*, vol.
789 70, pp. 1–8, 2015.
- 790 [50] S. Multon and F. Toutlemonde, “Effect of moisture conditions and transferts on
791 alkali silica reaction damaged structures,” *Cem. Concr. Res.*, vol. 40, pp. 924–934,
792 2010.
- 793 [51] N. Smaoui, M.-A. Bérubé, B. Fournier, and B. Bissonnette, “Influence of
794 Specimen Geometry, Orientation of Casting Plane, and Mode of Concrete
795 Consolidation on Expansion Due to ASR,” *Cem. Concr. Aggregates*, vol. 26, no. 2,
796 pp. 1–13, 2004.
- 797 [52] G. Wardeh and B. Perrin, “Freezing-thawing phenomena in fired clay materials
798 and consequences on their durability,” *Constr. Build. Mater.*, vol. 22, pp. 820–828,
799 2008.
- 800 [53] C. Larive, M. Joly, and O. Coussy, “Heterogeneity and anisotropy in ASR-affected
801 concrete –Consequences for structural assessment,” in *11th International*
802 *Conference on Alkali-Aggregate Reaction in Concrete*, 2000, pp. 969–978.
- 803 [54] S. Multon and F. Toutlemonde, “Effect of applied stresses on alkali-silica reaction-
804 induced expansions,” *Cem. Concr. Res.*, vol. 36, no. 5, pp. 912–920, 2006.
- 805 [55] C. F. Dunant and K. L. Scrivener, “Effects of uniaxial stress on alkali-silica
806 reaction induced expansion of concrete,” *Cem. Concr. Res.*, vol. 42, no. 3, pp.
807 567–576, 2012.
- 808 [56] B. P. Gautam, D. K. Panesar, S. A. Sheikh, and F. J. Vecchio, “Multiaxial
809 Expansion-Stress Relationship for Alkali Silica Reaction-Affected Concrete,” *ACI*
810 *Mater. J.*, vol. 114, no. 1, pp. 171–184, 2017.
- 811 [57] J. Liaudat, I. Carol, C. M. López, and V. E. Saouma, “ASR expansions in concrete
812 under triaxial confinement,” *Cem. Concr. Compos.*, vol. doi: 10.10, 2017.
- 813 [58] H. Kagimoto and M. Kawamura, “Measurements of strain and humidity within
814 massive concrete cylinders related to the formation of ASR surface cracks,” *Cem.*
815 *Concr. Res.*, vol. 41, pp. 808–816, 2011.
- 816 [59] R.-P. Martin, “Analyse sur structures modèles des effets mécaniques de la réaction
817 sulfatique interne du béton, Ph.D. thesis, LCPC Laboratoire Central des Ponts et
818 Chaussées Paris,” 2010.

- 819 [60] M. Salgues, “Modélisations des effets structuraux des réactions sulfatiques
820 internes et alcali-granulats : Application aux barrages en béton, PhD Thesis,
821 Université de Toulouse,” 2013.
- 822 [61] P. Morenon, S. Multon, A. Sellier, E. Grimal, F. Hamon, and E. Bourdarot,
823 “Impact of stresses and restraints on ASR expansion,” *Constr. Build. Mater.*, vol.
824 140, pp. 58–74, 2017.
- 825 [62] J. Simo, *Numerical analysis and simulation of plasticity*. Ciarlet, P., Lions, J.
826 (Eds.), Handbook of Numerical Analysis, vol. VI. North- Holland, 1998.
- 827 [63] S. Multon and A. Sellier, “Multi-scale analysis of alkali-silica reaction (ASR):
828 Impact of alkali leaching on scale effects affecting expansion tests,” *Cem. Concr.*
829 *Res.*, vol. 81, pp. 122–133, 2016.
- 830 [64] A. Leemann and P. Lura, “E-modulus of the alkali-silica-reaction product
831 determined by micro-indentation,” *Constr. Build. Mater.*, vol. 44, pp. 221–227,
832 2013.
- 833 [65] C. Zhang, L. Sorelli, B. Fournier, J. Duchesne, J. Bastien, and Z. Chen, “Stress-
834 relaxation of crystalline alkali-silica reaction products : Characterization by micro-
835 and nanoindentation and simplified modeling,” *Constr. Build. Mater.*, vol. 148, pp.
836 455–464, 2017.
- 837 [66] M. Kawamura and H. Fuwa, “Effects of lithium salts on ASR gel composition and
838 expansion of mortars,” *Cem. Concr. Res.*, vol. 33, pp. 913–919, 2003.
- 839 [67] S. Goto and D. M. Roy, “Diffusion of ions through hardened cement pastes,” *Cem.*
840 *Concr. Res.*, vol. 11, pp. 751–757, 1981.
- 841 [68] T. Kim, J. Olek, and H. Jeong, “Alkali–silica reaction: Kinetics of chemistry of
842 pore solution and calcium hydroxide content in cementitious system,” *Cem. Concr.*
843 *Res.*, vol. 71, pp. 36–45, 2015.
- 844 [69] A. Perruchot, P. Massard, and J. Lombardi, “Composition et volume molaire
845 apparent des gels Ca–Si, une approche expérimentale,” *C. R. Geosci.*, vol. 335, pp.
846 951–958, 2003.
- 847 [70] K. L. Scrivener and K. M. Nemati, “The percolation of pore space in the cement
848 paste/aggregate interfacial zone of concrete,” *Cem. Concr. Res.*, vol. 26, no. 1, pp.
849 35–40, 1996.
- 850 [71] K. L. Scrivener, A. K. Crumbie, and P. Laugesen, “The interfacial transition zone
851 (ITZ) between cement paste and aggregate in concrete,” *Interface Sci.*, vol. 12, no.
852 4, pp. 411–421, 2004.
- 853 [72] Y. Takahashi, S. Ogawa, Y. Tanaka, and K. Maekawa, “Scale-Dependent ASR
854 Expansion of Concrete and Its Prediction coupled with Silica Gel Generation and
855 Migration,” *J. Adv. Concr. Technol.*, vol. 14, pp. 444–463, 2016.
- 856 [73] T. M. Van Genuchten and D. Nielsen, “On describing and predicting the hydraulic
857 properties of unsaturated soils,” *Ann. Phys.*, vol. 3, no. 5, pp. 615–628, 1985.
- 858 [74] B. Zuber and J. Marchand, “Modeling the deterioration of hydrated cement

- 859 systems exposed to frost action - Part 1: Description of the mathematical model,”
860 *Cem. Concr. Res.*, vol. 30, no. 2000, pp. 1929–1939, 2000.
- 861 [75] A. Hillerborg, M. Mod er, and P.-E. Petersson, “Analysis of crack formation and
862 crack growth in concrete by means of fracture mechanics and finite elements,”
863 *Cem. Concr. Res.*, vol. 6, pp. 773–782, 1976.
- 864 [76] C. Comi, B. Kirchmayr, and R. Pignatelli, “Two-phase damage modeling of
865 concrete affected by alkali – silica reaction under variable temperature and
866 humidity conditions,” *Int. J. Solids Struct.*, vol. 49, no. 23–24, pp. 3367–3380,
867 2012.
- 868 [77] L. J. Stuble and S. Diamond, “Swelling Properties of Synthetic Alkali Silica Gels,”
869 *J. Am. Ceram. Soc.*, vol. 64, no. 11, pp. 652–655, 1981.
- 870 [78] H. W. Reinhardt and O. Mielich, “A fracture mechanics approach to the crack
871 formation in alkali-sensitive grains,” *Cem. Concr. Res.*, vol. 41, no. 3, pp. 255–
872 262, 2011.
- 873 [79] M. Prezzi, P. J. M. Monteiro, and G. Sposito, “The alkali-silica reaction, part I:
874 Use of the double-layer theory to explain the behavior of reaction-product gels,”
875 *ACI Mater. J.*, vol. 94, no. 94, pp. 10–17, 1997.
- 876 [80] R. Pignatelli, C. Comi, and P. J. M. Monteiro, “A coupled mechanical and
877 chemical damage model for concrete affected by alkali-silica reaction,” *Cem.*
878 *Concr. Res.*, vol. 53, pp. 196–210, 2013.
- 879