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ABSTRACT

The presented work is a step towards designing a numerical strategy capable of assessing the nocivity of a small defect in terms of its size and position in the structure with low computational cost, using only a mesh of the defect-free reference structure. We focus here on presenting two computational scenarios allowing to efficiently evaluate flaw criticality. These scenarios are considering either the effect of a fixed flaw for any evalutaion point in solid, or varying flaws on a fixed evaluation point.

1 Motivation, introduction and problem definition

The role played by defects in the initiation and development of rupture is crucial and has to be taken into account in order to realistically describe the behavior till complete failure. The difficulties in that context revolve around (i) the fact that the defect length scale is much smaller than the structure length scale, and (ii) the random nature of their position and size. Even in a purely deterministic approach, taking those defects into consideration by standard models imposes to resort to geometrical discretisations at the defect scale, leading to very costly computations and hindering parametric studies in terms of defect location and characteristics.

Our current goal is to design an efficient two-scale numerical strategy which can accurately predict the perturbation in terms of stress caused by an inhomogeneity in elastic (background) material. To make it computationally efficient, the analysis uses only a mesh for the defect-free structure, i.e. the mesh size does not depend on the (small) defect scale.

We consider a linearly elastic body occupying a smooth bounded domain $\Omega \subset \mathbb{R}^d$ (with the spatial dimensionality $d = 2$ or 3), whose boundary Γ is partitioned as $\Gamma = \Gamma_D \cup \Gamma_N$ support a prescribed traction $\bar{\mathbf{t}}$ and a prescribed displacement $\bar{\mathbf{u}}$, while a body force density $\mathbf{f}$ is applied in Ω. On the basis of this fixed geometrical and loading configuration, we consider two situations, namely (i) a reference solid characterized by a given elasticity tensor $\boldsymbol{\mathcal{C}}$, which defines the background solution $\mathbf{u}$, and (ii) a perturbed solid constituted of the same background material except for a small inhomogeneity whose material is characterized by $\boldsymbol{\mathcal{C}}^\star$, which defines a perturbed solution $\mathbf{u}_a$. The aim of this work is to formulate a computational approach allowing to treat case (ii) as a perturbation of the background solution (i), in particular avoiding any meshing at the small inhomogeneity scale. This will be achieved by applying known results on the asymptotic expansion of the displacement perturbation with respect to the small characteristic size a of the inhomogeneity to case (ii).

2 Small inhomogeneity asymptotics

The difference of the mentioned displacement fields gives a displacement perturbation $\mathbf{v}_a := \mathbf{u}_a - \mathbf{u}$. An asymptotic analysis of $\mathbf{v}_a$ with respect to the characteristic defect size a provides a way to evaluate the influence of the location, size, shape and material characteristics of defects on the solution $\mathbf{u}_a$. We focus in this work on the outer expansion given by [1, Thm. 11.4]

$$\mathbf{v}_a(\mathbf{x}) = -\nabla_{(1)}\boldsymbol{G}(\mathbf{z},\mathbf{x}) : \boldsymbol{\mathcal{A}}(\mathcal{B},\boldsymbol{\mathcal{C}},\Delta\boldsymbol{\mathcal{C}}) : \nabla\mathbf{u}(\mathbf{z})a^d + o(a^d), \qquad \mathbf{x} \neq \mathbf{z}, \tag{1}$$

where the leading contribution is given as multiplicative decompsition in terms of three key 'ingridients': elastostatic Green's tensor $\boldsymbol{G}$, elastic moment tensor (EMT) associated with the inhomogeneity $\boldsymbol{\mathcal{A}}$, and background strain (see [2]).

$\boldsymbol{G}(\boldsymbol{\xi},\mathbf{x})$ represents the displacement field being a response at $\boldsymbol{\xi} \in \Omega$ of the background body subjected to a unit point force applied at $\mathbf{x}$ with homogeneous boundary conditions. Using known symmetry relationship and decomposition $\boldsymbol{G} = \boldsymbol{G}_\infty + \boldsymbol{G}_c$ to full-space and complementary parts, respectively, we propose two distinct computational scenarios: (a) fixed inclusion location $\mathbf{z}$ and varying evaluation point $\mathbf{x}$, or (b) fixed evaluation point $\mathbf{x}$ and varying inclusion location $\mathbf{z}$.

3 Computational scenarios to compute inclusion perturbation

It turns out [2] that for scenario (a) one needs to solve the $d(d+1)/2$ problems (one for $\mathbf{u}$ and others for $\nabla\boldsymbol{G}_c$) to compute the perturbation anywhere in solid $\mathbf{x} \in \Omega$ for virutally any eliptic inclusion. That is, it's shape, size and material can vary with virtually no additional cost.
In the inverse approach, scenario (b), the evaluation point $\mathbf{x}$ will be a point from critical zone. The perturbation in $\mathbf{x}$ will be computed for virutally any eliptic inclusion positioned anywhere in solid, $\mathbf{z} \in \Omega$. (Note that $\mathbf{x} \neq \mathbf{z}$ for both scenarios).

Proposed computational scenarios can be used as numerical strategy for predicting the (stress) perturbation caused by small isolated inhomogeneities in elastic solids. Meshing the isolated inhomogeneity (with the attendant mesh preparation and computational costs) is avoided, the whole analysis relying on a mesh that is suitable for the defect-free configuration. Proposed approach will be illustrated on few numerical examples.

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