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Economies of scale: The physics basis

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Why is size so important? Why are "economies of scale" a universal feature of all flow systems, animate, inanimate, and human made? The empirical evidence is clear: the bigger are more efficient carriers (per unit) than the smaller. This natural tendency is observed across the board, from animal design to technology, logistics, and economics. In this paper, we rely on physics (thermodynamics) to determine the relation between the efficiency and size. Here, the objective is to predict a natural phenomenon, which is universal. It is not to model a particular type of device. The objective is to demonstrate based on physics that the efficiencies of diverse power plants should increase with size. The analysis is performed in two ways. First is the tradeoff between the "external" irreversibilities due to the temperature differences that exist above and below the temperature range occupied by the circuit executed by the working fluid. Second is the allocation of the fluid flow irreversibility between the hot and cold portions of the fluid flow circuit. The implications of this report in economics and design science (scaling up, scaling down) and the necessity of multi-scale design with hierarchy are discussed. *Published by AIP Publishing.*
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INTRODUCTION

Why is size so important? This fundamental question dominates theoretical thought throughout science, from animal design[1–12] and river basins[13–15] to vehicle technology,[1,16] logistics, and economics. The empirical observations are of the same kind, regardless of their realm of origin, animate or inanimate. Their summary is that the bigger are more "efficient" in their respective movement, or movers of animal weight, pumps of blood and water, movers of freight, manufacturer of goods, and communications. It is "cheaper" to move something as a component of a larger mover, as opposed to moving it alone, against its environment. This universal empirical fact (and source of wisdom in human design) is widely recognized as "economies of scale."

The expression "economies of scale" is part of common language, and it means savings (reductions) that are registered when a small job is handled in bulk by a bigger entity. This expression has an old history, and much of its use is rooted in the industrial revolution. For example, savings in effort and expenditure were made possible in the unloading of ships when many workers with sacks on their backs were replaced by one conveyor belt.

Why are economies of scale a universal phenomenon? To answer this question is the purpose of this article. To answer the question in a most fundamental way, we formulate the argument in physics, more precisely, in thermodynamics.

In constructing the physics argument for economies of scale, we benefit from recent advances in thermodynamics that unify the design and functioning of all moving (flow) systems. In every example, the physics is simple;[1,17] (i) the system moves against its environment because it is being pushed, (ii) the pushing is provided by power produced in "engines" (natural, including the human made), and (iii) the power is dissipated instantly during movement relative to a resisting environment.

In this physics framework, it is evident that the physics of "economies of scale" lies in the effect of size on the efficiency of the engine that drives the movement. This is the direction that the argument takes, and, for even greater clarity, the argument is constructed in terms of the simplest models of engines and efficiency concepts.

RESULTS

Performance data show that in eight classes of power generation installations the larger size is correlated with greater efficiency (Fig. 1).[18] The concept of efficiency here is used in the thermodynamics sense, as the ratio between the power output of an engine divided by the rate of heat input to that engine (or divided by the rate of fuel consumption). The efficiency of hydro-turbines exhibits the same size effect: greater efficiency is associated with greater power output, and greater power is associated with larger physical size (Fig. 2).[19] The design analysis of a condenser for a concentrated solar power plant showed that the efficiency loss decreases as the condenser face area increases (Fig. 3).[20] In ocean thermal power plants, the efficiency increases as the installed power output (base power) increases:[21] as shown in Fig. 4, the efficiency increases from 2.5% to 3.5% when the base power increases from 5 MW to 40 MW.

Figure 5 shows that the efficiency (η) of the engines for helicopters increases in proportion with the engine size (M_e, kg)

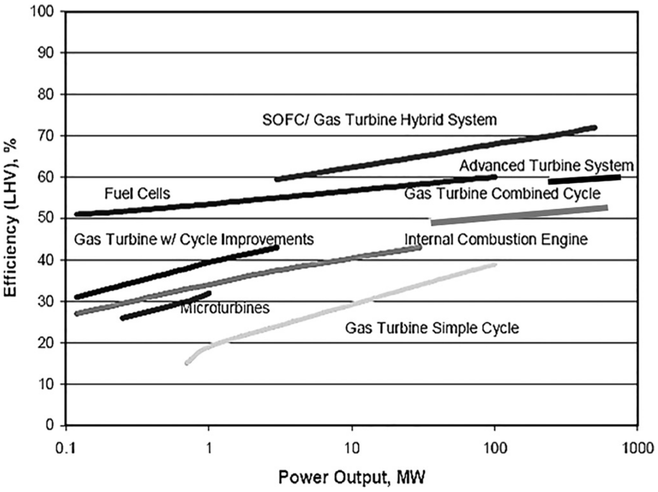

FIG. 1. Read in the horizontal direction, this chart shows the size effect on the efficiency of individual types of power plants. Read in the vertical direction, this figure shows the time arrow of technology evolution, which is toward more efficient power plant flow configurations.[1]

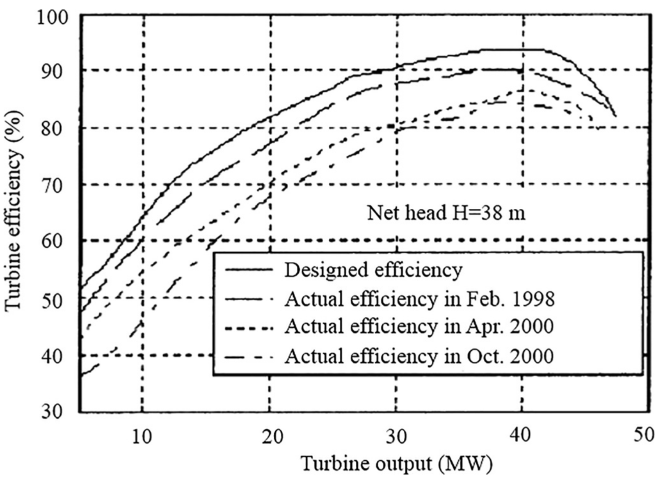

FIG. 2. The effect of size on turbine efficiency.[19]

raised to a power comparable with 1/4. The H factor is the heating value of the fuel. The data are from the helicopter models adopted throughout the history of helicopter evolution.[22] The data in Fig. 5 document the efficiencies of the power plants of all helicopter models, not the efficiencies of helicopters as vehicles.

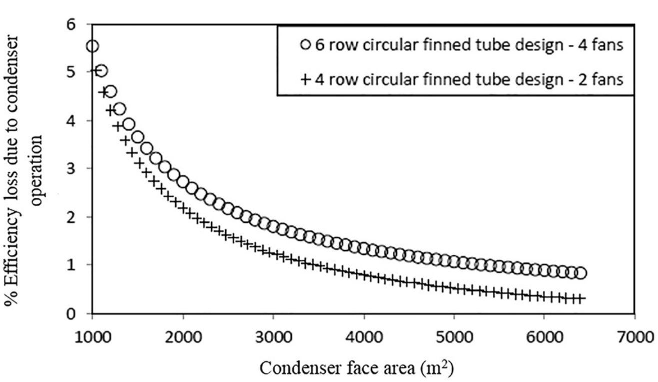

FIG. 3. The efficiency loss decreases as the condenser area increases.[20]

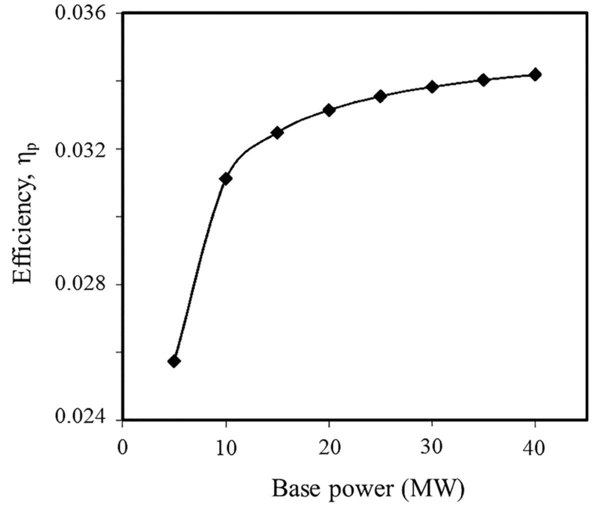

FIG. 4. The efficiency of ocean thermal power plants versus the base power.[21]

In summary, the phenomenon of economies of scale is present across the board. Even though in Fig. 2 the efficiency data are not aligned completely monotonically with size, they support the broad message of Figs. 1–5, which is that the bigger are more efficient. To uncover the physics basis for the phenomenon of economies of scale is the objective of this article.

Allocation of size

In Fig. 6(a), the power plant is modeled[23] as a closed system that operates in the steady state between two temperature reservoirs, T_H and T_L. The heat input is $\dot{Q}_H$ and the power output $\dot{W}$. In thermodynamics, there is already a sizeable literature that is based on simple models of this kind.[24] The irreversibility of power plant operation is made explicit in Fig. 6(b), where the power plant is viewed as a sandwich of three subsystems. The temperature difference $T_H - T_{HC}$ drives the heat input $\dot{Q}_H$, while the temperature difference $T_{LC} - T_L$ drives the rejected heat current $\dot{Q}_L$

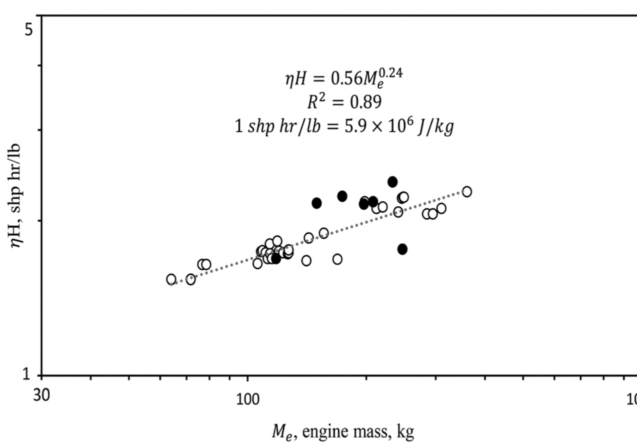

FIG. 5. The correlation between helicopter engine efficiency and engine size. In the indicated correlation, the military helicopter data (the black circles) were not included. If the military data are included, the correlation becomes $\eta H = 0.53 M_e^{0.25}$, *with* $R^2 = 0.79$.[22]

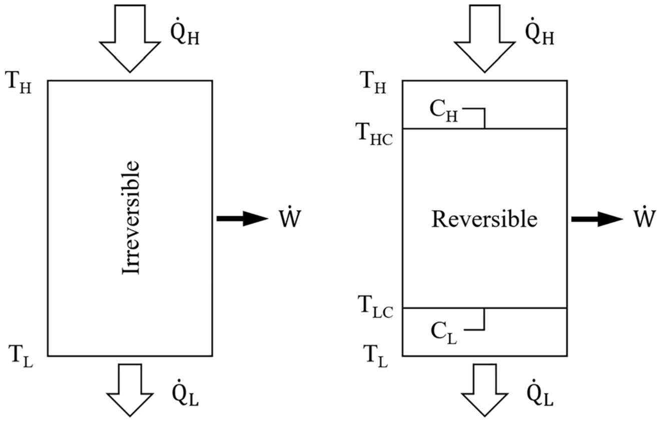

FIG. 6. (a) Power plant model as a closed system and (b) power plant model with two temperature gaps and a middle portion that operates reversibly.

$$\dot{Q}_H = C_H(T_H - T_{HC}), \tag{1}$$

$$\dot{Q}_L = C_L(T_{LC} - T_L). \tag{2}$$

C_H and C_L represent the heat transfer conductance of the two temperature gaps and are proportional to the respective sizes of the heat transfer surfaces that line each gap. The irreversibility of the power plant is concentrated in the two temperature gaps. For simplicity, it is assumed that the intermediate compartment is operating reversibly,

$$\frac{\dot{Q}_H}{T_{HC}} = \frac{\dot{Q}_L}{T_{LC}}. \tag{3}$$

The total size of the two heat transfer surfaces is represented by the sum of the heat transfer conductances as follows:

$$C = C_H + C_L. \tag{4}$$

The power output $\dot{W}$ is the same as the power production from the middle compartment

$$\dot{W}_C = \dot{Q}_H\left(1 - \frac{T_{LC}}{T_{HC}}\right). \tag{5}$$

The efficiency of the power plant, η, depends on the size of each heat exchanger, C_H and C_L,

$$\eta = \frac{\dot{W}_C}{\dot{Q}_H} = 1 - \frac{T_{LC}}{T_{HC}}, \tag{6}$$

where

$$T_{HC} = T_H - \frac{\dot{Q}_H}{C_H}, \tag{7}$$

$$T_{LC} = T_L - \frac{1 - \dfrac{\dot{Q}_H}{C_H T_H}}{1 - \left(1 + \dfrac{C_H}{C_L}\right)\dfrac{\dot{Q}_H}{C_H T_H}}. \tag{8}$$

Next, we account for the allocation of the conductance inventory (C) between the two ends of the power plant (C_H versus C_L) by introducing the fraction $x = C_H/C$, therefore

$$C_H = xC \text{ and } C_L = (1 - x)C. \tag{9}$$

Combining Eqs. (6)–(8), we find

$$\eta = 1 - \frac{T_L/T_H}{1 - \dfrac{\dot{Q}_H}{T_H C}\left(\dfrac{1}{x} - \dfrac{1}{1 - x}\right)}, \tag{10}$$

which shows that the efficiency is maximum at $x = 1/2$. In this configuration, where the thermal conductance inventory is allocated equally to the hot end and the cold end, the efficiency is

$$\eta = 1 - \frac{T_L/T_H}{1 - 4\dot{Q}_H/(T_H C)}. \tag{11}$$

Equation (11) shows that the maximized efficiency increases when the size C increases. This is illustrated in Fig. 7. The efficiency vs size is a concave curve: the efficiency increases at a decreasing rate as the size increases. Figure 7 also shows that η increases when T_H/T_L increase. In the limit of infinite size, the irreversibility of the two temperature gaps vanishes, and η approaches the Carnot efficiency, $1 - T_L/T_H$.

Allocation of irreversibility

Here, we consider the question of whether the uniform allocation of size ($C_H = C_L$) translates into uniform distribution of irreversibility between the hot and cold ends of the power plant. The rate of entropy generation in the hot compartment is

$$\dot{S}_{gen,H} = \dot{Q}_H\left(\frac{1}{T_{HC}} - \frac{1}{T_H}\right) = \dot{Q}_H\frac{\dot{Q}_H/C_H}{T_H(T_H - \dot{Q}_H/C_H)}. \tag{12}$$

The rate of the entropy generation in the cold compartment is

$$\dot{S}_{gen,L} = \dot{Q}_L\left(\frac{1}{T_L} - \frac{1}{T_{LC}}\right) = \dot{Q}_H\frac{\dot{Q}_H/C_H}{T_H(T_H - \dot{Q}_H/C_H)(T_H - 2\dot{Q}_H/C_H)}. \tag{13}$$

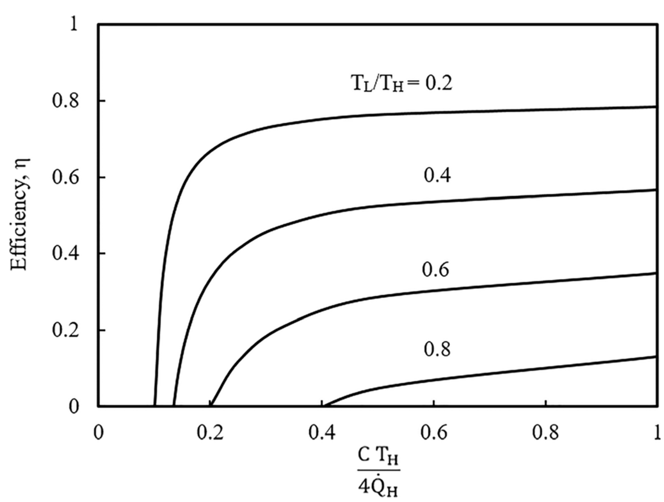

FIG. 7. The effect of the size of the total heat transfer surface (C) on the efficiency of the power plant.

The ratio

$$\frac{\dot{S}_{gen,H}}{\dot{S}_{gen,L}} = 1 - \frac{\dot{Q}_H}{T_H C_H} < 1 \tag{14}$$

shows that when the two compartments are of equal size, the cold compartment generates more entropy than the hot compartment. The total entropy generation rate

$$\begin{aligned}\dot{S}_{gen} &= \dot{S}_{gen,H} + \dot{S}_{gen,L}, \\ &= \frac{\dot{Q}_H/T_H}{\frac{CT_H}{4\dot{Q}_H} - 1}\end{aligned} \tag{15}$$

shows that if the size increases indefinitely, the irreversibility of the power plant vanishes, in agreement with the conclusion reached under Eq. (11).

Allocation of fluid flow volume

Next, we consider the fluid flow irreversibility associated with power generation. The model is presented in Fig. 8. This time, instead of heat transfer across finite temperature differences (Fig. 6), we isolate the effect of fluid flow with friction or pressure drop. This effect is presented at the hot and cold ends of the power plant, in the two ducts that line the heat transfer surfaces discussed in "Results" section.

The irreversible power plant model (Fig. 8(b)) is a sandwich of three compartments: the duct at the hot end, the middle compartment that is assumed to operate reversibly, and the duct located at the cold end. The reversible compartment generates the power output $\dot{W}_C = \dot{Q}_H(1 - T_L/T_H)$. The actual power output is $\dot{W}$, and it is smaller than $\dot{W}_C$,

$$\dot{W} = \dot{W}_C - \dot{W}_H - \dot{W}_L \tag{16}$$

because of the mechanical power needed in order to push the working fluid (of flow rate $\dot{m}$) through the hot and cold ducts.

The model for estimating $\dot{W}_H$ and $\dot{W}_L$ is based on Fig. 8(c). The duct has the flow cross section A_f, wetted perimeter p, hydraulic diameter $D_h = 4A_f/p$, swept length L, contact (heat transfer) surface area A, and volume

$$V = LA_f. \tag{17}$$

For illustration, assume first that the flow is in the fully developed laminar regime. In this case, the pressure drop along the duct is

$$\Delta P_{lam} = \frac{4L}{D_h} f \frac{1}{2} \rho U^2, \tag{18}$$

where $f = Po/Re$, $Re = UD_h/\nu$, $U = \dot{m}/(\rho A_f)$, and Po is the Poiseuille constant, which depends on the shape of A_f cross section.[25] Then, Eq. (18) becomes

$$\Delta P_{lam} = \frac{2L\, Po\, \nu\, \dot{m}}{D_h^2\, A_f}. \tag{19}$$

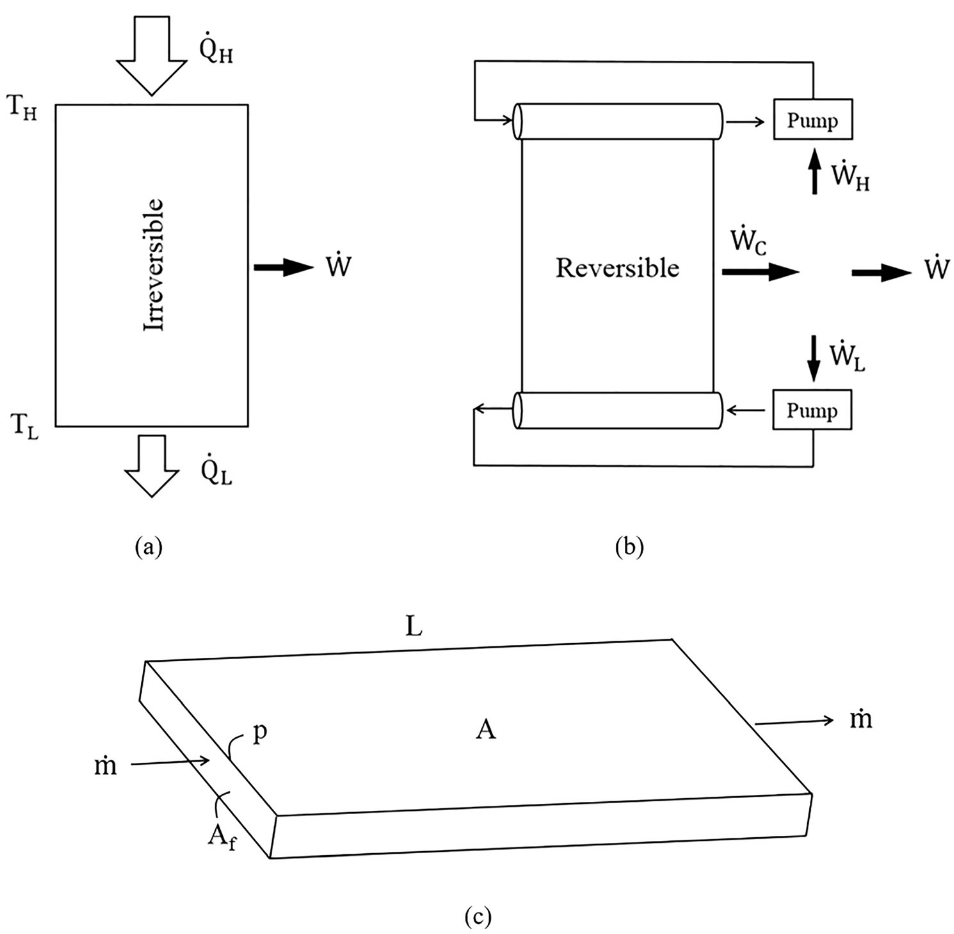

FIG. 8. (a) Power plant as a closed system operating irreversibly; (b) model with fluid flow irreversibilities at the hot end and the cold end; and (c) the scales of the three dimensional duct model.

On the other hand, if the flow is in fully developed and fully rough turbulent regime, then Eq. (18) holds with f ≅ constant, and yields

$$\Delta P_{turb} = \frac{2Lf\dot{m}^2}{\rho\, D_h\, A_f{}^2}. \tag{20}$$

Next, we use scale analysis to recognize the L and A_f scales

$$L \sim A^{1/2},\ A_f \sim D_h L \tag{21}$$

such that Eqs. (19) and (20) become

$$\Delta P_{lam} \sim 2Po\,\nu\,\dot{m}\,\frac{A^3}{V^3}, \tag{22}$$

$$\Delta P_{turb} \sim \frac{2}{\rho} f\ \dot{m}^2\,\frac{A^{5/2}}{V^3}. \tag{23}$$

The similarities between Eqs. (22) and (23) are evident. The pumping power dissipated driving fluid flow through the duct flow is $\dot{W} = \dot{m}\Delta P/\rho$, which means

$$\dot{W}_{lam} \sim \frac{2}{\rho}\,Po\,\nu\,\dot{m}^2\,\frac{A^3}{V^3} = \frac{K_{lam}}{V^3}, \tag{24}$$

where $K_{lam} \sim \frac{2}{\rho}\,Po\,\nu\,\dot{m}^2\,A^3$, and

$$\dot{W}_{turb} \sim \frac{2}{\rho^2} f\dot{m}^3\,\frac{A^{5/2}}{V^3} = \frac{K_{turb}}{V^3}, \tag{25}$$

where $K_{turb} \sim \frac{2}{\rho^2} f\,\dot{m}^3\,A^{5/2}$.

In summary, regardless the flow regime, the pumping power required by one duct has the form

$$\dot{W}_{duct} \sim \frac{K}{V^3} \tag{26}$$

for which the expressions for K_{lam} and K_{turb} are given under Eqs. (24) and (25). Equation (26) holds for each of the two ducts shown in Fig. 8(b),

$$\dot{W}_H \sim \frac{K_H}{V_H^3}, \tag{27}$$

$$\dot{W}_L \sim \frac{K_L}{V_L^3}, \tag{28}$$

where V_H and V_L are the respective duct volumes, the sum of which is the total flow volume constraint,

$$V = V_H + V_L,\ \text{constant}. \tag{29}$$

Defining the duct volume allocation fraction $y = V_H/V$, such that $V_L/V = 1 - y$, the total pumping power becomes

$$(\dot{W}_H + \dot{W}_L)V^3 \sim \frac{K_H}{y^3} + \frac{K_L}{(1-y)^3}. \tag{30}$$

This quantity is minimum when

$$y = \frac{1}{1 + (K_L/K_H)^{1/4}} \tag{31}$$

and has the scale

$$(\dot{W}_H + \dot{W}_L)_{min} \sim (K_H{}^{1/4} + K_L{}^{1/4})^4 V^{-3}. \tag{32}$$

This new result reinforces the conclusion that the phenomenon of economies of scale is predictable, because the minimized pumping power loss [Eq. (32)] decreases very sharply as the size (V) increases. In this configuration, the duct volume V is allocated according to the fraction

$$\frac{V_H}{V_L} = \frac{y}{1-y} = \left(\frac{K_H}{K_L}\right)^{1/4}. \tag{33}$$

The irreversibility, or the dissipation of useful power, is allocated according to the same fraction

$$\frac{\dot{W}_H}{\dot{W}_L} = \frac{K_H/V_H^3}{K_L/V_L^3} = \left(\frac{K_H}{K_L}\right)^{1/4}. \tag{34}$$

In conclusion, the fluid fraction irreversibility is allocated between the two ends of the power plant in the same proportion as the available duct volume. Which end has more duct volume (and pumping power) depends on the ratio K_H/K_L. Four different cases of power plants are considered in Table I. For example, if at the hot end the $\dot{m}$ stream is supercritical water at $T_H = 600\,°C$ and $P_H = 27$ MPa, and if the flow regime is turbulent with $f \sim 10^{-2}$, then

$$K_H \sim (3.4 \times 10^{-6}\dot{m}^3\,A^{5/2})Wm^9, \tag{35}$$

where $\dot{m}$ and A are expressed in kg/s and m^2. If at the cold end the $\dot{m}$ stream is saturated steam at 40 °C, and the flow is turbulent with f $\sim 10^{-2}$, then

$$K_L \sim (2 \times 10^{-8}\dot{m}^3\,A^{5/2})Wm^9. \tag{36}$$

Next, assume that the contact (heat transfer) surface A is allocated equally between the hot and cold ends, so that the efficiency is maximum. Then, the duct volume allocation ratio (V_H/V_L) is approximately 3.6. In power plants with superheated steam, the hot-end volume should be roughly 6

TABLE I. Calculated flow volume ratios for various power plants.

Case	Temperature (°C)	Pressure (MPa)	Density (kg/m^3)	State	K × 10^{-6} [Wm^9]	V_H/V_L
1	600	27	76.56	Supercritical	3.4	3.60
2	500	25	89.8	Supercritical	2.5	3.32
3	600	10	26.06	Superheated	29	6.17
4	500	8	24.27	Superheated	34	6.39
Cold end	40	0.0074	992.17	Saturated steam	0.02	…

times larger than the cold-end volume, cf. cases 3 and 4 in Table I.

DISCUSSION

We end the analysis with a few observations regarding the model used to unveil the physics origin of the phenomenon of economies of scale. One reviewer questioned the inclusion of internal combustion engines in Fig. 1 because IC engines do not have high temperature heat exchangers, unlike in the simple model used in the body of our paper. This is incorrect, as thermodynamics. Of course the engine has a "high temperature," and a low temperature as well, otherwise it would not be an engine. The exergy input to the boiler comes from combustion, and it implies the existence of a "high temperature." In thermodynamics, the concept of exergy transfer by heat transfer (E_Q) is based on three features of the device that experiences exergy transfer:[23] a heat interaction, Q, a high temperature, T_f, and a low temperature, T_L,

$$E_Q = Q\left(1 - \frac{T_L}{T_f}\right). \tag{37}$$

The exergy "input" to the boiler is the Carnot work that is associated with the heat input and the two temperatures across a Carnot engine. Alternatively, if we know the exergy input, the heat input, and the low temperature (as in the reviewer's "combustion" example), then we know the high temperature T_f, which is the "effective flame temperature of combustion as an exergy source" defined in Ref. 23. The reviewer also noted that the internal combustion engine does not have a high temperature heat exchanger. In reality, the high temperature heat exchanger is the piston cylinder that serves as a combustion chamber, and inside this chamber the burning fuel + air mixture and the cold air intake constitute a "direct contact" heat exchanger. Furthermore, the larger size of this direct-contact heat exchanger goes with a more powerful and more efficient engine. This is the size effect that the authors of the handbook[18] documented, cf. Fig. 1 in this paper.

The reviewer also observed that a gas turbine power plant (or a car engine) does not have a "low temperature heat exchanger." The low temperature heat exchanger is the atmosphere, "the big sewer in the sky," which receives the hot exhaust from the engine, cools it by mixing (by direct contact, again), and makes it available as cold air at the inlet to the compressor of the gas turbine power plant, and at the inlet to the cylinder of the car engine.

The fact that in some engines some components are not made by humans does not mean that such features do not exist. The discipline of thermodynamics allows us to see all the components, even in devices where nature provides for free the missing hardware. In the earth-size engine that drives the atmospheric and oceanic circulation, all the hardware components are "missing."[1]

CONCLUSIONS

In this report, we showed that the phenomenon of economies of scale is predictable from pure physics, and consequently it is present in all flow systems that experience evolutionary changes in their configurations toward greater global performance. We demonstrated the physics basis by considering a power plant model with heat transfer and fluid flow irreversibility distributed between the hot end and the cold end. The total heat transfer surface inventory (the thermal conductance, C) and the total fluid flow volume (V) were fixed. We showed that the overall efficiency increases at decreasing rate as the thermal conductance inventory increases. This means that the predicted efficiency-size curve must be concave, in accord with the empirical data exhibited in Figs. 1–5. We also found that the total pumping power loss decreases proportionally with V^{-3} as total flow volume increases. The convexity of the power loss vs size curve means that the corresponding effect of V on efficiency is a η-V curve that is concave.

We chose the simplest possible models for how heat and fluid flow through a closed system that generates power. The simplest models are in Figs. 6 and 8 and are complicated enough so that they capture the physics, which is the effect of size on efficiency. With these models we do not mean to suggest that we are representing one of the power plants of Fig. 1. The objective of theory is to show how to predict the natural effect in the simplest manner, from the point of view from which the phenomenon is most visible.

The chief conclusion is about the physics basis of the economies of scale phenomenon: the bigger should function with fewer losses *per unit* of size. It is not about predicting the efficiency vs. size curve of a particular flow system that generates power. All the examples compiled in Fig. 1 share the features that were included in the models of Figs. 6 and 8. They have fluid currents that flow through small or large duct cross sections, and heat and mass currents that pass through small or large transfer surfaces.

The fundamental path outlined in this report deserves to be explored in greater detail, especially in applied physics, animal design, economics and technology evolution.[1,12,13] A primary feature of the present work is the ability to predict how the performance changes with scaling up and scaling down the design of flow system. In brief, the flow design is an architecture that changes predictively according to the size of the system. Magnifying or miniaturizing a known design is not the way to discover the proper configuration and performance of a larger and, respectively, smaller flow system.

Another application of the present work is this: if the natural evolutionary trend is toward architectures that flow more easily and larger architectures are more efficient, then why are not all the flow systems evolving toward being larger and larger?

The reason is that all flow in nature is on an area or in volume, as in Fig. 8(c): one stream in, one stream out, and a finite volume that is bathed fully, i.e., vascularized. The flow is between one point and an infinity of points (area, volume). Even though the bigger streams are more efficient carriers, the finite-size area or volume cannot be bathed everywhere by big channels. Small channels are necessary in order to bathe the interstices completely. To sweep the area, few large movers and many small movers must flow together, hierarchically,[1] because this is how movement is facilitated the most on the area or volume.

In sum, the thermodynamics presented in this paper showed that the economies of scale phenomenon is a fundamental feature of all flow (moving) systems, animate, inanimate, and human made. The size effect on efficiency manifests itself in other features of design, distinct from the power generation considered in our paper. For example, the rate of heat loss from a furnace, a cooking vessel, or the body of an animal is smaller *per unit* of body size (mass, volume).

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