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ABSTRACT: Numerical modeling of the behavior of geotechnical structures subjected to cyclic loading requires the consideration of two main aspects: the constitutive relationship and numerical strategy. The constitutive model has to be representative of the clay behavior, whereas the numerical strategy should be efficient in order to reduce the computation time. To simulate undrained triaxial cyclic tests on clay, a method of time homogenization is applied to a bounding surface plasticity model. This method of homogenization is based upon splitting time into two separate scales. The first scale relates to the period of cyclic loading and the second to the characteristic time of the material. Simulations of undrained triaxial cyclic tests on normally consolidated clay under one-way cyclic loading are carried out. The performance of time homogenization is numerically validated. The sensitivity of the time increment is also investigated.

INTRODUCTION

Structures such as wind power plants, offshore installations, embankments, railways and tunnels are subjected to a large number of loading cycles. The phenomenon of fatigue in soils is identified and characterized (Andersen 2009), but design tools for these geotechnical problems are missing. Two main problems have to be considered.

First, the constitutive model has to be able to reproduce with good accuracy the cyclic behavior of soil. During the last decades, multi-surface models have been developed. Most of them belong to kinematic hardening plasticity theory (Mroz 1967) or to bounding surface plasticity (Dafalias and Popov 1975). The latter receives significant attention because of its simplicity and efficiency. This type of model is selected for this study and described in the following section.

Second, the conventional simulation of a structure subjected to a large number of cycles is time consuming. Thus different strategies have to be developed which could require only the simulation of a few cycles, the other cycles being treated differently according to the strategies considered (e.g. Wichtmann 2005).

In this paper, one of these strategies, called time homogenization, is selected. Time homogenization, as spatial homogenization, is derived from mathematical perturbation theory. Bensoussan et al. (1978) followed by Sanchez-Palencia (1980) introduced these methods for periodic structures. This paper presents the principles upon which time homogenization is based and the conclusions are applied to the outlined model in order to simulate undrained triaxial cyclic tests.

CONSTITUTIVE MODEL

The constitutive model refers to the theory of bounding surface plasticity. The implementation of the model is based on the works of Dafalias and Herrmann (1986) and Manzari and Nour (1997). This model is a bounding surface version of the Modified Cam-Clay with Hooke's elasticity. It assumes two surfaces: the yield surface (called bounding surface) and the subyield surface (called loading surface). They are homothetic with the respect to a projection center. Figure 1a represents both surfaces and the projection center corresponds to the origin of the coordinates. The current stress point σ defines the loading surface. When the current stress point is on the loading surface, an "image" stress $\bar{\sigma}$ is defined as the projection of the current stress σ on the bounding surface with respect to the projection center (see Fig. 1a).

The relation between image and current stresses is called the mapping rule. The flow rule is respected and the plasticity parameter is given by:

$$\lambda = \frac{1}{\bar{K}_P}\left(\frac{\partial F}{\partial \bar{\sigma}}\dot{\bar{\sigma}}\right) = \frac{1}{K_P}\left(\frac{\partial F}{\partial \bar{\sigma}}\dot{\sigma}\right) \tag{1}$$

where F is the potential surface. The plastic modulus at the current stress point K_P is a function of the plastic modulus at the image stress point $\bar{K}_P$ and of the distance between bounding and loading surfaces. Based on the relation proposed by Manzari and Nour (1997), the plastic modulus is given by:

$$K_P = \bar{K}_P + H_0 \cdot \frac{(1+e_0)}{\rho} \cdot p'^{\,3}_{c0} \cdot (\beta - 1) \tag{2}$$

where β is equal to the ratio $p'_{c0} / p'_{c\sigma}$ (see Fig. 1a). When the current stress point σ is on the bounding surface ($\beta = 1$), Eq. (2) is reduced to the same form as the conventional Modified Cam-Clay model.

The constitutive model has the parameters of the Modified Cam-Clay model (Young's modulus E, Poisson's ratio ν, critical state slope M, consolidation pressure p'_{c0}, $\rho = \lambda - \kappa$ with λ slope of the normal consolidation line and κ slope of the swelling line, and initial void ratio e_0), and one additional parameter, namely the hardening parameter H_0. Figure 1b represents the simulation of an undrained triaxial test on a normally consolidated clay under one-way cyclic loading in (p',q) plan. The values of the model parameters are given in Table 1 and H_0 is taken equal to 1000.

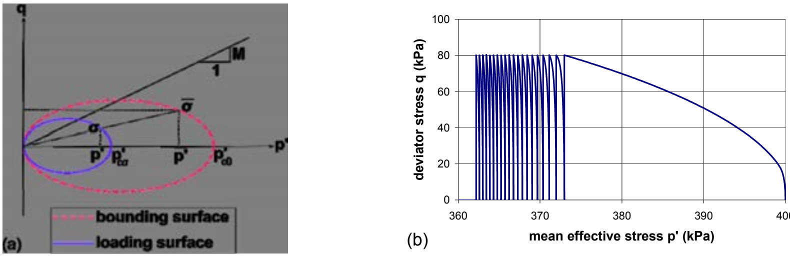

FIG. 1. (a) Representation of bounding and loading surfaces in (p', q) plan. (b) Simulation of an undrained triaxial test on a normally consolidated clay under one-way cyclic loading in (p',q) plan.

TIME HOMOGENIZATION

Fatigue of a material subjected to cyclic loading evolves slowly in comparison to the duration of one single loading cycle. It cannot be detected by considering only few cycles, but by considering a large number of cycles. Therefore the evolution during one single load cycle needs to be described by an accurate time scale, whereas a coarser one is enough for the fatigue evolution.

The aim of time homogenization is to separate the effects that take place at the two time scales in order to use small time increments only when it is necessary and so reduce time computation. To do so, we define the time characteristic of the cycles τ_0 (the period) and the time characteristic of the fatigue phenomenon t_r which depends on material properties. On a graph representing the evolution of a variable related to fatigue (e.g. permanent strain or pore pressure) as a function of the time, t_r can be defined as the time corresponding to the intersection of the line of the asymptotic behavior and the tangent at the origin. τ_0 and t_r can be characterized by two different units. The ratio of these units is noted ζ From this small positive ratio and the natural time scale describing the long-term fatigue, a second time scale describing the loading can be derived: $\tau = t/\zeta$ The time homogenization method, as it is explained further, consists in approximating displacements and stresses by using asymptotic expansions with respect to the ratio ζ Since we consider the first order of the asymptotic expansions, the smaller the ratio ζ the better the results.

Although the selected constitutive model is a time-independent model which does not involve any notions of time, we can still apply this method by considering a fictitious time related to the number of cycles. Mathematical proof is given by Guennouni (1988), who considers an elastoplastic problem as the limit of a series of elastoviscoplastic problems. As shown by Guennouni (1988) and Yu (2002), the homogenization method makes possible the separation of the two time scales so that the original initial-boundary value problem (Pb) can be divided into two problems (Pb*) and (Pb**).

$$
(\text{Pb})\quad \begin{cases} \sigma_{ij,j} + b_i(x) = 0 \\ \sigma_{ij} = C_{ijkl} : (\varepsilon_{kl} - \varepsilon^p_{kl}) \\ \dot{\varepsilon}^p_{kl} = \lambda \dfrac{\partial F}{\partial \sigma_{kl}} \\ \dot{p}'_c = p'_c \dfrac{1+e}{\lambda - \kappa} \dot{\varepsilon}^P_v \end{cases} \quad \text{and} \quad \begin{cases} \varepsilon_{ij} = \dfrac{(u_{i,j} + u_{j,i})}{2} \\ u_i(x, t=0) = \tilde{u}_i(x) \\ u_i = \bar{u}_i(x,t) \\ \sigma_{ij} n_j = f_i(x,t) \end{cases} \tag{3}
$$

where b_i corresponds to volumetric forces; σ_{ij}, ε_{ij} and ε^P_{ij} are respectively the components of stress, strain and plastic strain tensors; C_{ijkl} the components of elastic tensor; u_i the component of displacement vector. The displacement, stress, plastic strain and hardening variable are approximated by asymptotic expansions in the form of:

$$
\alpha^{\varsigma}(x,t) = \sum_{M=0,1,..} \varsigma^M \alpha_M(x,t,\tau) \tag{4}
$$

where α_M τ-periodic. Using Eq. (3) and (4) and considering the first-order term of the asymptotic expansions, it appears that the plastic strains and the hardening variable are τ-independent, so that their evolution is noticed only in the long term. Moreover, the global behavior of the material can be divided into the non-oscillatory long-term behavior and the oscillatory short-term behavior. The first part is defined as the average over a cycle of the first-order term of the asymptotic expansions; the second part as the remaining term.

$$
\begin{aligned} u_0(x,t,\tau) &= \langle u_0(x,t,\tau)\rangle + \chi_0(x,t,\tau) \\ \sigma_0(x,t,\tau) &= \langle \sigma_0(x,t,\tau)\rangle + \phi_0(x,t,\tau) \\ \varepsilon_0(x,t,\tau) &= \langle \varepsilon_0(x,t,\tau)\rangle + \psi_0(x,t,\tau) \end{aligned} \qquad \text{where } \langle \bullet \rangle = \frac{1}{\tau_0} \int_0^{\tau_0} \bullet \, d\tau \tag{5}
$$

The non-oscillatory and oscillatory terms are respectively the response fields of the problems (Pb*), called the macro-chronological problem, and (Pb**), called the micro-chronological problem. It is worth noting that (Pb**) is a linear elastic problem, which can be solved at the beginning of the calculation. The constitutive relation of (Pb*) is called the homogenized constitutive relation and depends on the behavior of the material. Eq. (6) and (7) give both problems (Pb*) and (Pb**) as follows:

$$
(\text{Pb*})\quad \begin{cases} \langle \sigma_{0ij} \rangle_{,j} + b_i(x) = 0 \\ \langle \sigma_{0ij} \rangle_{,t} = C_{ijkl} : (\langle \varepsilon_{0kl} \rangle_{,t} - \varepsilon^p_{0kl,t}) \\ \varepsilon^p_{0kl,t} = \langle \lambda \dfrac{\partial F}{\partial \sigma_{0kl}} \rangle \\ p'_{c0,t} = p'_{c0} \dfrac{1+e}{\lambda - \kappa} \varepsilon^P_{0v,t} \end{cases} \quad \text{and} \quad \begin{cases} \langle u_{0i} \rangle (x, t=0) = \tilde{u}_i(x) \\ \langle u_{0i} \rangle = \langle \bar{u}_i(x,t,\tau) \rangle \\ \langle \sigma_{0ij} \rangle n_j = \langle f_i(x,t,\tau) \rangle \end{cases} \tag{6}
$$

$$
(\text{Pb**})\quad \begin{cases} \phi_{0ij,j}(x,t,\tau) = 0 \\ \phi_{0ij,\tau}(x,t,\tau) = C_{ijkl} : \psi_{0kl,\tau}(x,t,\tau) \end{cases} \quad \text{and} \quad \begin{cases} \chi_{0i}(x, t=\tau=0) = 0 \\ \chi_{0i}(x,t,\tau) = \bar{u}_i(x,t,\tau) - \langle \bar{u}_i(x,t,\tau) \rangle \\ \phi_{0ij} n_j = f_i(x,t,\tau) - \langle f_i(x,t,\tau) \rangle \end{cases} \tag{7}
$$

NUMERICAL VALIDATION

The model with homogenization is implemented in a FORTRAN routine which is used to simulate undrained triaxial tests and to estimate the reduction of time computation. An undrained triaxial test on a normally consolidated clay under one-way cyclic loading (q = 80 kPa) is assumed for the numerical validation. The permanent strain is defined as the strain remaining after a cycle. The values of the model parameters corresponding to a kaolinite clay are summarized in Table 1. The first ten cycles are simulated conventionally because of the important rate of permanent strains at the beginning of the calculation (see Fig. 1b).

Table 1. Model parameters.

E (kPa)	ν	M	p'_{c0} (kPa)	κ	λ	e_0	H_0
30000	0.3	0.7	400	0.04	0.19	1.5	1000; 3000;10000

The time increment corresponds to N_c cycles. Simulations are carried out for three values of N_c: 50, 100 and 200. The homogenized calculation is about N_c times faster than the conventional one. Figures 2(a-b) show the influence of N_c over 2000 cycles. The maximal error for permanent strain (83%) due to homogenization appears for N_c = 200 at the 210th cycle. However, the error is reduced to 14% after 2000 cycles and becomes less than 1.2% after 30000 cycles (see Fig. 2c).

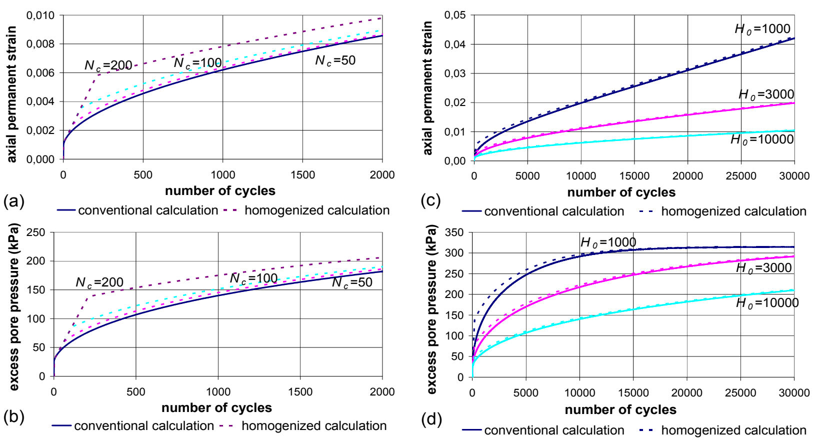

FIG. 2. Comparison of conventional and homogenized calculations: (a) permanent strain / (b) excess pore pressure for H_0 = 1000 and N_c = 50, 100, 200; (c) permanent strain / (d) excess pore pressure for H_0 = 1000, 3000, 10000 and N_c = 200.

Figures 2(c-d) compare the conventional and homogenized calculations with different values of the hardening parameter. The characteristic time t_r depends on this parameter, as shown by the different curvature of the curves, and it explains why the calculation with $H_0 = 1000$ is at first the most affected by the homogenization strategy.

CONCLUSIONS

The example of cyclic modeling outlined in this paper shows to what extent time homogenization can be used to efficiently and accurately model cyclic behavior of soils. When applied to a bounding surface plasticity model, this strategy can significantly improve computational efficiency and guarantees over a large number of cycles that the same results will be obtained as by conventional calculations. Particular attention should be paid to the choice of the time increment. The paper presents the numerical validation for the time homogenization technique applied to a bounding surface model. For further studies, it will be interesting to validate the approach by using real experimental testing results on soil samples as well as boundary value problems.

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